Color superconductivity and compact stars

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References

- I.A.S., M. Hanuske & M. Huang, *hep-ph/0303027*
**Birth of compact stars**

![Diagram showing the process from red giant to supernova explosion and remnant.]

Typical parameters:

<table>
<thead>
<tr>
<th>Mass</th>
<th>Radius</th>
<th>Temperature</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 1.4 , M_{\odot}$</td>
<td>$\sim 10 , \text{km}$</td>
<td>$20 - 40 , \text{MeV}$</td>
<td>$\lesssim 10 \rho_0$</td>
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</table>
Quark stars?

CHANDRA X-Ray Observatory (April 10, 2002)
Cosmic X-rays Reveal Evidence For New Form Of Matter

RX J1856.5-3754 [Drake et al., 02]  3C58 [Slane et al., 02]

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Quark stars?

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The New York Times

“Observations of two stars, one unusually small and the other unusually cold, have led astronomers to think they are seeing evidence of a new form of matter and a new kind of star, one possibly made of elementary particles known as quarks and denser than any cosmic object other than a black hole.”

Were quark stars observed? — Perhaps not yet ...

[Walter et al., 02], [Yakovlev et al., 02], [Zane et al., 03]
Weakly interacting quark matter

The property of asymptotic freedom: \( \alpha_s(\mu) \ll 1 \) for \( \mu \gg \Lambda_{QCD} \)

[Gross & Wilczek,73], [Politzer,73]

- Dense quark matter is weakly interacting [Collins & Perry,75]
- “Squeezing” quark matter

- Realistic densities in compact stars: 
  \( \rho \approx 10\rho_0 \), where \( \rho_0 \approx 0.15 \text{ fm}^{-3} \),
  (corresponding coupling \( \alpha_s \approx 1 \))
Basics of color superconductivity

Asymptotic density ($\mu \gg \Lambda_{QCD}$):

- $\alpha_s(\mu) \ll 1$ (weak coupling)
- One-gluon interaction is dominant
- Color $\bar{3}_a$ channel is attractive (!)

\[ p \quad k \]
\[ \frac{1}{\kappa} \]
\[ = \bar{3}_a + 6_s \]
\[ -p \quad -k \]

- BCS mechanism for quarks leads to color superconductivity
- By using Pauli principle ($s = 0$):

\[ N_f = 2 : \varepsilon_{ij} \varepsilon^{ab} \langle (\psi^i_a)^T C \gamma^5 \psi^j_b \rangle \neq 0 \]
\[ N_f = 3 : \sum_{I=1}^{3} \varepsilon_{ijI} \varepsilon^{abi} \langle (\psi^i_a)^T C \gamma^5 \psi^j_b \rangle \neq 0 \]

[Son], [Schafer et al.], [Hong et al.], [Pisarski et al.], [Shovkovy et al.] (1999)
Properties of 2SC ground state

(up & down quarks only)

- Chiral symmetry $SU(2)_L \times SU(2)_R$ is intact
- Color symmetry is broken (by Anderson-Higgs mechanism):
  $SU(3)_c \rightarrow SU(2)_c$
  - color Meissner effect (for 5 gluons)
  - low energy $SU(2)_c$ gluodynamics (decoupled)
- Modified electromagnetic $U(1)_{\tilde{e}m}$ and modified $U(1)_{\tilde{B}}$ survive
  - no electromagnetic Meissner effect
  - no superfluidity
- Approximate $U(1)_A$ is broken $\rightarrow$ light pseudo-NG boson
- Parity is preserved
Signatures of CSC in compact stars

Color superconductivity $\rightarrow$ gap in quasiparticle spectrum

- Thermodynamic properties (equation of state)
  - mass-radius relation [Alford&Reddy,02], [Lugones&Horvath,02]
  - internal star structure [Baldo et al.,02], [Shovkovy et al.,03]

- Transport properties (conductivities, viscosities, mean free paths)
  - cooling rate [Page et al.,02], [Shovkovy&Ellis,02]
  - r-mode instability [Madsen,99]
  - glitches (crystalline phase) [Alford et al.,00]

- Other properties
  - magnetic field generation/penetration [Alford et al.,00]
  - rotational vortices [Iida&Baym,02]
Neutrality vs. color superconductivity

- The “best” 2SC phase appears when $n_d \approx n_u$,
- but neutral matter appears when $n_d \approx 2n_u$
- Electrons do not help (!):

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3}\mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

Thus, $n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$

- Cooper pairing with a mismatch between Fermi surfaces of pairing quarks:

$$\mu_d - \mu_u = \mu_e$$

Gaps: $(\Delta + \mu_e/2)$ and $(\Delta - \mu_e/2)$

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Neutral quark phases

- Locally neutral phases:
  - Normal quark matter
  - gapless 2SC matter [Shovkovy&Huang,03]

- Globally neutral mixed phases [Glendenning,92], e.g., 2SC+NQ (?)

\[ \rho_{e}^{(MP)} = \chi_{B}^{A}\rho_{e}^{(A)}(\mu, \mu_{e}) + (1 - \chi_{B}^{A})\rho_{e}^{(B)}(\mu, \mu_{e}) = 0 \]

where

\[ \chi_{B}^{A} \equiv \frac{V^{(A)}}{V^{(A)} + V^{(B)}} \text{ is the volume fraction of phase A} \]
Gibbs construction \((2\text{SC}+\text{NQ})\)

- Mechanical equilibrium:
  \[ P^{(2\text{SC})}(\mu, \mu_e) = P^{(\text{NQ})}(\mu, \mu_e) \]

- Chemical equilibrium:
  \[ \mu = \mu^{(2\text{SC})} = \mu^{(\text{NQ})}, \]
  \[ \mu_e = \mu_e^{(2\text{SC})} = \mu_e^{(\text{NQ})} \]

- From the condition of neutrality
  \[ \chi_{2\text{SC}}^{\text{NQ}} = \frac{\rho_e^{(\text{NQ})}}{\rho_e^{(\text{NQ})} - \rho_e^{(2\text{SC})}}, \]

Energy density: \( \varepsilon^{(\text{MP})} = \chi_{2\text{SC}}^{\text{NQ}} \varepsilon^{(2\text{SC})}(\mu, \mu_e) + (1 - \chi_{2\text{SC}}^{\text{NQ}}) \varepsilon^{(\text{NQ})}(\mu, \mu_e) \)
Coulomb forces and surface tension

- Extra surface and Coulomb energies per unit volume
  \[ E_S \simeq C_S^{(\text{geom})} \frac{\sigma}{R}, \quad E_C \simeq C_C^{(\text{geom})} \left( \rho_e^{(A)} - \rho_e^{(B)} \right)^2 R^2 \]

- Minimizing the sum with respect to \( R \), one gets
  \[ E_{C+S} \simeq (8 \text{ MeV fm}^{-3}) \left( \frac{\sigma \rho_e^{(A)} - \rho_e^{(B)}}{\sigma_0 \rho_e^{(0)}} \right)^{2/3}, \quad \text{("slabs")} \]
  where \( \sigma_0 = 50 \text{ MeV fm}^{-2} \) and \( \rho_e^{(0)} = 0.4e \text{ fm}^{-3} \)

- Thickness of "slabs"
  \[ a \simeq (9.4 \text{ fm}) \left( \frac{\sigma}{\sigma_0} \right)^{1/3} \left( \frac{\rho_e^{(0)}}{\rho_e^{(A)} - \rho_e^{(B)}} \right)^{2/3}, \]

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Coulomb and surface effects in 2SC+NQ matter

Coulomb effects are easy to estimate, while surface tension is usually not well known.

There are three possible cases:

- Low surface tension \((\sigma \lesssim 20 \text{ MeV fm}^{-2})\):
  little effect; mixed phase survives (“slabs” with \(a \simeq 10 \text{ fm}\))

- Intermediate values of surface tension \((20 \lesssim \sigma \lesssim 50 \text{ MeV fm}^{-2})\):
  phase transition occurs at higher densities, \(3\rho_0 \lesssim \rho_B \lesssim 5\rho_0\)

- Large values of surface tension \((\sigma \gtrsim 50 \text{ MeV fm}^{-2})\):
  homogeneous phase is more favorable than mixed phase

Similar estimates are valid for hadron-quark mixed phases

[Heiselberg et al.,01], [Alford et al.,01]
Hadronic matter

- At low densities $\sim \rho_0$ quarks are confined
- Some hadronic description is required
- We use hadronic chiral $SU(3)_L \times SU(3)_R$ model [Papazoglou, 98], [Papazoglou, 99], [Hanauske, 00]
  - nonlinear realization of $SU(3)_L \times SU(3)_R$
  - spontaneous symmetry breaking
  - small explicite symmetry breaking
  - QCD motivated dilaton field is included
- Model describes well hadronic masses, finite nuclei, hypernuclei and neutron star properties
Hybrid matter

Hadronic phase $\rightarrow$ Hadron-quark MP $\rightarrow$ 2SC+NQ quark MP

- Star crust matter:
  $\rho_B \leq 0.08 \text{ fm}^{-3}$ [Baym et al., 71], [Negele & Vautherin, 73]

- Hadronic matter:
  $0.08 \leq \rho_B \leq 1.49 \text{ fm}^{-3}$

- Hadron-quark mixed phase:
  $1.49 \leq \rho_B \leq 2.56 \text{ fm}^{-3}$

- 2SC+NQ quark mixed phase:
  $\rho_B \geq 2.75 \text{ fm}^{-3}$

$\triangle$-point is a triple point (!)
Star structure is determined by metric that satisfies Tolman-Oppenheimer-Volkoff equations

**Input:** equation of state $P(\varepsilon)$

- $\square$ – beginning of hadron-quark MP
- $\triangle$ – beginning of 2SC+NQ quark MP

- $\varepsilon$ and $\rho_B$ have jumps at the triple point

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Compact star structure

- $\epsilon_c = 210$ MeV/fm$^3$ – pure hadronic star;
- $\epsilon_c = 370$ MeV/fm$^3$ – hybrid star without quark core;
- $\epsilon_c = 500$ MeV/fm$^3$ – hybrid star with a quark core;
- $\epsilon_c = 1392$ MeV/fm$^3$ – largest mass star with parameters:
  $M_{\text{max}} = 1.81M_\odot$
  $\rho_c = 7.58\rho_0$
  $R = 10.86$ km

There are no stars with $378 \leq \epsilon_c \leq 415$ MeV/fm$^3$
Stars heavier than “○-stars” may have strange matter in their cores
Summary

- Realistic EoS of nonstrange hybrid baryon matter is constructed
- Charge neutrality and $\beta$-equilibrium are taken into account; they play very important role
- Two-flavor color superconducting matter appears naturally as a component of 2SC+NQ mixed phase
- Construction of 2SC+NQ mixed phase is very stable: volume fractions of components change little with changing density
- The first example of a triple point is obtained and studied
- Sharp interface between the two mixed phases is observed; this is smoothed over distances of about 10 fm
**Outlook**

- Generalization including strange quarks is needed
- All kinds of hybrid star properties should be studied
  - neutrino emissivity and mean free path
  - cooling rates of mixed phases
  - magnetic properties
- Surface tension effects and screening of Coulomb forces should be addressed
- Studies of color superconductivity in rotating stars are of interest
- Search for signatures of color superconductivity in compact stars should be made systematic