CHIRAL MATTER:
from Quark Gluon Plasma to Topological Semimetals

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(Quasi-) Relativistic Matter

- **Early Universe**
  (extremely high temperature $> 10^{15}$ K)

- **Heavy-ion collisions**
  (high temperature $\leq 4 \times 10^{12}$ K)

- **Super-dense matter in compact stars**
  (high densities $\leq 10^{17}$ kg/m$^3$)

- **Magnetospheres of magnetars**
  (electron-positron plasma at temperatures $\leq 10^{11}$ K)

- **Electron plasma in Dirac/Weyl (semi-)metals**
  (chiral quasiparticle plasma at temperatures $\leq 10^2$ K)

- **Other: cold atoms, superfluid $^3$He-A, etc.**
  (chiral quasiparticles at temperatures $\sim 10^{-3}$ K)
ANOMALOUS MATTER

Chirality

• Only *massless* Dirac/Weyl fermions have a well-defined chirality $(\gamma^5 \psi = \pm \psi)$:

  ![Right-handed](image)

  **Right-handed** (spin parallel to momentum)

  ![Left-handed](image)

  **Left-handed** (spin opposite to momentum)

• The chirality of *massive* Dirac fermions is *almost* well-defined in the *ultra-relativistic* regime*

  – High temperature: $T \gg m$

  – High density: $\mu \gg m$

*Note: like the particle spin, chirality is a quantum property*
Anomalous chiral matter

- Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ existing on *macroscopic* time/distance scales

- The spacetime dynamics of $n = n_R + n_L$ and $n_5 = n_R - n_L$ is governed by continuity equations

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\frac{\partial n_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_m n_5$$

where the chirality flip rate: $\Gamma_m \propto \alpha^2 T (m/T)^2$

- Chiral anomaly can produce *macroscopic* effects in plasmas
Anomalous effects

• **Theory**: Many *macroscopic* chiral anomalous effects were proposed

  - Some are triggered by an external magnetic field
    - Chiral magnetic effect
    - Chiral separation effect
    - Chiral magnetic wave
    - Negative magnetoresistance
    - ...

  - Others are triggered by vorticity
    - Chiral vortical effect
    - Chiral vortical wave
    - ...


CHIRAL ANOMALOUS EFFECT

\[ \langle \hat{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2} \mu \quad & \quad \langle \hat{j} \rangle = \frac{e^2\vec{B}}{2\pi^2} \mu_5 \]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]
Landau levels

- Landau energy levels at $m = 0$

$$E_n^\pm = \pm \sqrt{2n|eB| + p_z^2}$$

where $n = k + \frac{1}{2} + \text{sgn}(eB)s_z$

- Lowest Landau level is spin polarized

$$E_0^\pm = \pm p_z \quad (k = 0, \ s_z = -\frac{1}{2})$$

- Higher Landau levels ($n \geq 1$) are twice as degenerate and non-polarized:

  (i) $k = n$ \& $s = -\frac{1}{2}$

  (ii) $k = n - 1$ \& $s = +\frac{1}{2}$

Chiral Separation Effect ($\mu \neq 0$)

- Spin polarized LLL is chirally asymmetric
  - states with $p_3 < 0$ (and $s = \downarrow$) are R-handed
  - states with $p_3 > 0$ (and $s = \downarrow$) are L-handed

i.e., a nonzero axial current is induced

$$\langle \hat{j}_5 \rangle = - tr[\gamma y^5 S(x, x)] = - \frac{eB}{2\pi^2 \mu}$$
Chiral Magnetic Effect ($\mu_5 \neq 0$)

Assume a *transient* state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL ($s=\downarrow$ for particles of a *negative* charge):

- Some **R-handed** states ($p_3 < 0$ \& $E < \mu_5$) are occupied
- Some **L-handed** states ($p_3 < 0$ \& $|E| < \mu_5$) are empty (i.e., holes with $p_3 > 0$)

CME current: $\langle \vec{j} \rangle = -tr [\vec{\gamma} S(x, x)] = \frac{e^2 B}{2\pi^2} \mu_5$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]
HEAVY-ION COLLISIONS
Rotating & magnetized QGP created at RHIC/LHC

Electromagnetic fields are induced by the currents of passing charged ions

Vorticity estimate: \( \omega \sim 9 \times 10^{21} \text{s}^{-1} (\sim 10 \text{ MeV}) \)

Magnetic field estimate: \( B \sim 10^{18} \text{ to } 10^{19} \text{ G} (\sim 100 \text{ MeV}) \)
Source of chirality in QCD

- Chiral charge can be produced by topological configurations in QCD

\[ \frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3 x \ F_{\mu\nu}^{a} \tilde{F}_{\mu\nu}^{a} \]

- A random fluctuation with nonzero chirality should produce

\[ N_R - N_L \neq 0 \implies \mu_5 \neq 0 \]

- The latter leads to an electric CME current

\[ \langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5 \]
Dipole CME

- Dipole pattern of *charged particle correlations* in heavy-ion collisions

\[
\langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}) \rangle > 0 \quad \& \quad \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}) \rangle < 0
\]

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]

Correlations of same & opposite charge particles:

\[
\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle > 0
\]

\[
\langle \cos(\phi_\alpha^\pm + \phi_\beta^\pm - 2\Psi_{RP}) \rangle < 0
\]

Large background effects!

\[\text{[Belmont & Nagle, PRC 96, 024901 (2017)]}\]
Isobar collisions

Utilize collisions of isobars, e.g.,

\[
\begin{align*}
{^{96}_{40}}\text{Zr} + {^{96}_{40}}\text{Zr} & \text{ vs. } {^{96}_{44}}\text{Ru} + {^{96}_{44}}\text{Ru} \\
{^{130}_{52}}\text{Te} + {^{130}_{52}}\text{Te} & \text{ vs. } {^{130}_{56}}\text{Ba} + {^{130}_{56}}\text{Ba}
\end{align*}
\]

[Voloshin, PRL 105, 172301 (2010)]
[Deng et al. PRC 94, 041901(R) (2016)]

Isobar collisions (theory)

Isobar collisions (experiment)

- Isobar run was completed by STAR in May 2018
- \( \approx 3.8 \) billion collisions of \(^{96}\text{Ru} + ^{96}\text{Ru}\) and \(^{96}\text{Zr} + ^{96}\text{Zr}\) at \(\sqrt{s} = 200\) GeV
- Blind analysis by five groups of the STAR Collaboration
- Report announced on Aug. 31, 2021 (online event @ BNL)
- Paper published on Jan. 3, 2022


Influence of signal and backgrounds, the STAR Collaboration performed a blind analysis of a large data sample of approximately 3.8 billion isobar collisions of \(^{44}\text{Ru} + ^{44}\text{Ru}\) and \(^{40}\text{Zr} + ^{40}\text{Zr}\) at \(\sqrt{s_{NN}} = 200\) GeV. Prior to the blind analysis, the CME signatures are predefined as a significant excess of the CME-sensitive observables in Ru + Ru collisions over those in Zr + Zr collisions, owing to a larger magnetic field in the former. A precision down to 0.4% is achieved, as anticipated, in the relative magnitudes of the pertinent observables between the two isobar systems. Observed differences in the multiplicity and flow harmonics at the matching centrality indicate that the magnitude of the CME background is different between the two species. No CME signature that satisfies the predefined criteria has been observed in isobar collisions in this blind analysis.
DIRAC & WEYL SEMIMETALS

[Gorbar, Miransky, Shovkovy, Sukhachov, Electronic Properties of Dirac and Weyl Semimetals (World Scientific, Singapore, 2021)]
Dirac/Weyl fermions

- Electron quasiparticles with a wide range of properties are possible

- They may even have the emergent spinor structure of massless Weyl fermions,

\[ H_W \approx \pm v_F (\hat{\sigma} \cdot \vec{k}) \]

Such nodes are not uncommon!

Na\textsubscript{3}Bi, Cd\textsubscript{3}As\textsubscript{2}, ZrTe\textsubscript{5}, TaAs, NbAs, …

[Liu et al., Science 343, 864 (2014)]
[Neupane et al., Nature Commun. 5, 3786 (2014)]
[Li et al., Nature Physics 12, 550 (2016)]
[S.-Y. Xu et al., Science 349, 613 (2015)]
[S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
[S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]
Relativistic-like band crossing

Do energy levels cross? Or do they repel?

A generic 2-band Hamiltonian reads

\[ H_\mathbf{k} = a_\mathbf{k} + \mathbf{b}_\mathbf{k} \cdot \mathbf{\sigma} \implies E_\mathbf{k} = a_\mathbf{k} \pm \sqrt{(b_\mathbf{k})^2} \]

The bands cross when \[ \mathbf{b}_\mathbf{k} = 0 \]

These 3 equations can be solved by adjusting \( \mathbf{k} \) in 3D

Emergent chirality in solids

Near a band crossing (e.g., $\mathbf{k} \approx \mathbf{k}_+$)

$$H_k = a_{k+} + \left( \nabla_k a_k \cdot \delta \mathbf{k} \right) + \sum_{i,j} \sigma_i b_{ij} \delta k_j$$

cone tilting

After an orthogonal transformation

$$b_{ij} \equiv \partial b_i / \partial k_j \rightarrow \hbar v_i \delta_{ij}$$

Assuming isotropy & a suitable reference point,

$$H_k = \pm v_F (\mathbf{\sigma} \cdot \mathbf{k})$$

which is the Weyl Hamiltonian

The chirality is defined by

$$\lambda = \text{sign} [ \det (b_{ij}) ]$$
Weyl quasiparticles

• The quasiparticle eigenstates for Weyl Hamiltonian $H_\lambda = \lambda v_F (\vec{k} \cdot \vec{\sigma})$ are
  $$\psi_k^\lambda = \frac{1}{\sqrt{2 \sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}}} \left( \frac{\nu_F k_z + \lambda \epsilon_k}{\nu_F k_x + i \nu_F k_y} \right)$$

• The quasiparticle energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$ is relativistic-like

• Function $k \rightarrow \psi_k^\lambda$ has a nontrivial topology

• Consider adiabatic evolution of the wave function from $\psi_k$ to $\psi_{k+\delta k}$:
  $$\langle \psi_k | \psi_{k+\delta k} \rangle \approx 1 + \delta \vec{k} \cdot \langle \psi_k | \nabla_k | \psi_k \rangle \approx e^{i a_k \cdot \delta \vec{k}}$$

where $a_k = -i \langle \psi_k | \nabla_k | \psi_k \rangle$ is the Berry connection
For Weyl eigenstates, the Berry curvature is

\[ \Omega_k \equiv \nabla_k \times a_k = \lambda \frac{\vec{k}}{2k^3} \]

The Chern number (topological charge)

\[ C = \frac{1}{2\pi} \oint \Omega_k \cdot dS_k = \frac{\lambda}{2\pi} \oint \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin \theta \, d\theta \, d\phi = \lambda \]

In a solid state material, the Brillouin zone is compact.

A closed surface around a node at \( \vec{k}_0 \) is also a closed surface around the rest of the Brillouin zone.

Thus, Weyl fermions come in pairs of opposite chirality.


[ Morimoto & Nagaosa, Scientific Reports 6, 19853 (2016)]
Idealized Dirac and Weyl model

- Low-energy Hamiltonians of a Dirac and Weyl materials

\[ H = \int d^3 r \overline{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{p}) - (\vec{b} \cdot \vec{\gamma}) \gamma^5 + b_0 \gamma^0 \gamma^5 \right] \psi \]

**Dirac** (e.g., Na$_3$Bi, Cd$_3$As$_2$, ZrTe$_5$)

**Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe$_2$)
Anomalous effects in semimetals

- Observable properties of Dirac/Weyl semimetals are sensitive to (i) the chiral anomaly, (ii) the values of $b_0$ and $\tilde{b}$, and (iii) nontrivial topology

- Partial list of potential anomalous effects:
  - Negative magnetoresistance ($\rho_\parallel$ decreasing with $B$)
  - **New types of collective modes** (anomalous Hall waves, pseudo-magnetic helicons, chiral zero sound, etc.)
  - Anomalous thermoelectric effects (e.g., $\vec{J}_Q \propto \vec{b} \times \vec{E}$ and $\vec{J}_Q \propto \vec{b} \times \nabla T$)
  - Strain/torsion induced CME ($\vec{J} \propto u_{33} \vec{B}$ and $\vec{J} \propto \mu \vec{B}_5$)
  - Strain/torsion dependent conductivity/resistance
  - Quantum oscillations in thin films [$T \propto v_F/\left(\mu b\right)$]
  - Strain/torsion induced quantum oscillations (pseudo-Landau levels)
  - Nonlocal anomalous transport
NEGATIVE MAGNETORESISTANCE & MORE

Image credit [Zhang et al., Nat. Commun. 7, 10735 (2016)]
Steady CME current

- Homogeneous chiral plasma:
  \[
  \frac{\partial n_5}{\partial t} + \nabla \times J_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \frac{n_5}{\tau_{ch}}
  \]

- Steady state \((\tau_{ch} \sim 1 \text{ ps to } 1 \text{ ns})\)
  \[
  n_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} \tau_{ch} \implies \mu_5 = \frac{n_5}{\chi_5} \approx \frac{3v^3 n_5}{T^2 + \mu^2 / \pi^2}
  \]

- The CME current
  \[
  J_i = \frac{e^2}{2\pi^2} \mu_5 B_i = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{ch} \frac{B_i B_k}{\chi_5} E_k \implies \sigma_{CME}^{\parallel} = \left(\frac{e^2}{2\pi^2}\right)^2 \tau_{ch} \frac{B^2}{\chi_5}
  \]
  i.e.,
  \[
  \rho_{\text{total}}^{\parallel} = \frac{1}{\sigma_0 + a(T)B^2}
  \]

[Son & Spivak, Phys. Rev. B 88, 104412 (2013)]
Negative Magnetoresistance

- Experimental confirmation

\[
\rho_{\text{total}}^\parallel = \frac{1}{\sigma_0 + a(T)B^2}
\]

Dirac semimetals:

[Li et al., Nat. Mater. 12, 550 (2016)]
[Xiong et al., Science 350, 413 (2015)]
[Li et al., Nat. Commun. 6, 10137 (2015)]
[Li et al., Nat. Commun. 7, 10301 (2016)]

Weyl semimetals:

[Huang et al., Phys. Rev. X 5, 031023 (2015)]
[Zhang et al., Nat. Commun. 7, 10735 (2016)]
[Hirschberger et al., Nat. Mater. 15, 1161 (2016)]
[Wang et al., Phys. Rev. B 93, 121112 (2016)]
[Li et al., Front. Phys. 12, 127205 (2017)]
Chiral charge pumping (theory)

- Weyl semimetal TaAs
  - $\vec{B} \neq 0$ & oscillating $\vec{E} \parallel \vec{B}$
- The nonlinear contribution to chiral charge-pumping conductivity
  \[
  \delta \sigma_{\text{ch} \text{NL}} = i \frac{9 \alpha^2 e^5 v^3}{8 h^2 \omega^3} \left( \frac{\vec{E}_{\text{pump}} \cdot \vec{B}}{B} \right)^2 B
  \]
- The reflection coefficient
  \[
  R(T) = \left| \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}} \right|^2 \quad \text{where} \quad \epsilon = \epsilon_{\infty} + i \frac{\sigma}{\omega \epsilon_0}
  \]
Chiral charge pumping (data)

- Experimental setup
  - Chiral charge relaxation time
    \[ 1 \text{ ns} \ll \tau_{ch} < 77 \text{ ns} \]

- Measurements:

  [Jadidi et al., Phys. Rev. B 102, 245123 (2020)]
Nonlocal anomalous transport

- **Theory**
  
  \[ \alpha_{NL} = \frac{R_{NL}}{R_{L}} \propto e^{-L/L_V} \]

  [Parameswaran, Grover, Abanin, Pesin, Vishwanath, PRX 4, 031035 (2014)]

- **Experiment** (challenge: Ohmic diffusion)
  
  [Zhang et al., Nat. Commun. 8, 13741 (2017)]

- **Measurements:**
  
  \[ L_V \sim 2 \, \mu m \]
CHIRAL STRAINTRONICS

[Zubkov, Annals Phys. 360, 655 (2015)]
[Pikulin, Chen, Franz, Phys. Rev. X 6, 041021 (2016)]
Pseudo-electromagnetic fields

- **Strains** modify the low-energy effective Weyl Hamiltonian

\[ H = \int d^3 r \bar{\psi} \left[ -iv_F (\vec{\gamma} \cdot \vec{p}) - (\vec{b} + \vec{A}_5) \cdot \vec{\gamma} \gamma^5 + (b_0 + A_{5,0}) \gamma^0 \gamma^5 \right] \psi \]

via the emergent **chiral gauge fields** are

\[
\begin{align*}
A_{5,0} &\propto b_0 \left| \vec{b} \right| \partial || u || \\
A_{5,\perp} &\propto \left| \vec{b} \right| \partial || u || \\
A_{5,||} &\propto \alpha \left| \vec{b} \right|^2 \partial || u || + \beta \sum_i \partial_i u_i
\end{align*}
\]

leading to the **pseudo-EM** fields

\[
\begin{align*}
\vec{B}_5 &= \nabla \times \vec{A}_5 \\
\vec{E}_5 &= -\nabla A_0 - \partial_t \vec{A}_5
\end{align*}
\]

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Continuity relations (with $\vec{B}_5$ & $\vec{E}_5$)

- Naïve continuity relations from chiral kinetic theory:
  \[
  \frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2 \hbar^2 c} \left[ (\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5) \right]
  \]
  ✔
  \[
  \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2 \hbar^2 c} \left[ (\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B}) \right]
  \]
  ❌

- Extra Bardeen-Zumino (Chern-Simons) terms are needed, i.e.
  \[
  \delta \rho = -\frac{e^3}{2\pi^2 \hbar^2 c^2} \left( (\mathbf{b} + \vec{A}_5) \cdot \mathbf{B} \right)
  \]
  \[
  \delta \mathbf{j} = -\frac{e^3}{2\pi^2 \hbar^2 c^2} \left( \mathbf{b}_0 + A_{5,0} \right) \mathbf{B} + \frac{e^3}{2\pi^2 \hbar^2 c^2} \left[ (\mathbf{b} + \vec{A}_5) \times \mathbf{E} \right]
  \]

  - Electric charge is conserved ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - No CME in equilibrium ($\mu_5 = -eb_0$)
INSTRUCTIVE EXAMPLE: COLLECTIVE MODES

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 97, 121105(R) (2018)]
Maxwell equations

Faraday’s law:
\[ \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \]
or in momentum space:
\[ \frac{c}{\omega} \vec{k} \times \vec{E} = \vec{B} \]

Ampere-Maxwell’s law:
\[ \nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \]
or in momentum space:
\[ \frac{c}{\omega} \vec{k} \times \left( \frac{c}{\omega} \vec{k} \times \vec{E} \right) = - \left( 4\pi \frac{i}{\omega} \vec{J} + \vec{E} \right) \]

Gauss’s law constrains \( \vec{E} \) as follows:
\[ i\vec{k} \cdot \vec{E} = 4\pi \rho \]

Anomalous contributions enter here
Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ \( k=0 \):

\[
\omega_l = \Omega_e, \quad \omega_{tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta \Omega_e}{\Omega_e}}
\]

where the Langmuir frequency is

\[
\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2} \left( \mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right)}
\]

and

\[
\delta \Omega_e = \frac{2e\alpha \nu_F}{3\pi c \hbar^2} \left\{ 9\hbar^2 b_{\perp}^2 + \left[ \frac{2\nu_F}{\Omega_e^2} \left( B_0 \mu + B_{0,5} \mu_5 \right) \right] \right. \\
- 3\hbar b_{\parallel} - \frac{\nu_F \hbar^2}{4T} \sum_{\lambda=\pm} B_{0,\lambda} F \left( \frac{\mu \lambda}{T} \right) \right\}^{1/2}
\]

[Go-bar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 95, 115202 (2017)]
Strain-induced helicons ($B = 0$)

- Strain-induced pseudo-magnetic field $B_{0,5}$ leads to

$$\omega_h \bigg|_{B_0 \to 0, \mu \to 0} \xrightarrow{b_0 \to -\mu_5/e} \frac{eB_{0,5}c^3\hbar^2\pi v_F^2k^2}{\pi \hbar^2c^2\Omega_e^2\mu_5 + 2B_{0,5}e^4v_F^2b_\parallel} + O(k^3)$$

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B 95, 115422 (2017)]

- Properties:
  - Gapless electromagnetic wave propagates in metals **without magnetic field**!
  - Chiral shift modifies effective helicon dispersion
  - In equilibrium, i.e., $\mu_5 = -eb_0$, the term linear in the wave vector is **absent**
Summary

• Chiral anomaly can have macroscopic implications in relativistic plasmas

• (Dipole) chiral magnetic effect can be seen via charged particle correlations in heavy-ion collisions

• Latest isobar measurements are promising but inconclusive (more studies are underway)

• Chiral anomaly can be realized and tested in Dirac/Weyl semimetals

• Chiral charge, which is relatively long-lived, can be optically pumped and manipulated (promising new technologies)