Chiral anomalous magnetospheres of magnetars

Igor Shovkovy

CHIRAL PLASMA

Anomalous plasma

• Chiral relativistic plasma may allow $n_L \neq n_R$ to persist on macroscopic time/distance scales

• Slow evolution of $n = n_R + n_L$ and $n_5 = n_R - n_L$ is controlled by the continuity equations

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

and

$$\frac{\partial n_5}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_m n_5$$

where the chirality flip rate: $\Gamma_m \propto \alpha^2 T(m/T)^2$

• Chiral anomaly can produce macroscopic effects in plasma
Chiral plasmas in nature

- **Heavy-ion collisions** (high temperature)

- **Super-dense matter in compact stars** (high density)
  [Yamamoto, Phys. Rev. D 93, 065017 (2016)]

- **Early Universe** (high temperature)

- **Electron plasma in Dirac/Weyl (semi-)metals**
  [Gorbar, Miransky, Shovkovy, Sukhachov, Electronic Properties of Dirac and Weyl Semimetals (World Scientific, Singapore, 2021)]

- **Other**: cold atoms, superfluid $^3$He-A, etc.
  [Volovik, JETP Lett. 105, 34 (2017)]

- **Magnetospheres of magnetars** [Gorbar & Shovkovy, arXiv:2110.11380]
  (electron-positron plasma at moderately high temperature)
Neutron stars

- **Neutron stars** are laboratories of matter under extreme conditions

- **Prediction**
  

- **Observation**
  
  [Hewish, Bell, Pilkington, Scott & Collins, *Nature* 217, 709 (1968)]

- **Pulsars** are neutron stars that are
  
  - rapidly rotating ($P \sim 1$ ms to 10 s)
  
  - strongly magnetized ($B \sim 10^8$ to $10^{15}$ G)

- **Pulsar radiation** is beamed along the magnetic field direction (the “lighthouse” effect)
Pulsars in $P-\dot{P}$ plane

- Characteristic age
  \[ \tau \approx \frac{P}{2\dot{P}} \]

- Spin-down luminosity
  \[ -\dot{E} \approx 4\pi^2 I \frac{\dot{P}}{P^3} \]

- Characteristic magnetic field
  \[ B \approx 3 \times 10^{19} \left( \frac{P \dot{P}}{s} \right)^{1/2} \text{G} \]

MAGNETOSPHERES

Image credit: Aurore Simonnet, Sonoma State University
Pulsar electrodynamics (VDM)

- Vacuum dipole model (VDM) \((\rho = 0 \& J = 0 \text{ outside the star})\)
- Stellar interior (good conductor):
  \[
  \vec{E}'_{in} = \vec{E}_{in} + \frac{\vec{\Omega} \times \vec{r}}{c} \times \vec{B}_{in} = 0
  \]
- Fields outside the pulsar are
  \[
  \vec{B} = \frac{B_0 R^3}{2r^3} (3(\hat{m} \cdot \hat{r})\hat{r} - \hat{m})
  \]
  \[
  \vec{E} = \ldots \quad [\text{see Deutsch, Ann. Astrophys. 18, 1 (1955)}]
  \]
  where \(\vec{m}\) is the magnetic moment and \(\Omega\) is the angular frequency
- There is a nonzero charge density and a strong electric field on the surface
  \((E_{surf} \sim \Omega R B_0 \sim 10^{12} \text{ to } 10^{15} \text{ V/m})\)
Pulsar electrodynamics (VDM)

- Charged particles
  
  i. Pulled up from the surface ($\vec{E} \neq 0$)
  
  ii. Move along curved trajectories ($\vec{B} \neq 0$)
  
  iii. Produce curvature radiation
  
  iv. $\gamma$-quanta produce $e^+ e^-$ pairs

  \[ l_\gamma \approx \frac{2R_c}{15} \frac{B_c m_e}{B \varepsilon_\gamma} \]

  v. Secondary particles produce synchrotron & curvature radiation

- **End result:** (I) Magnetized vacuum is nontransparent for photons with $\varepsilon_\gamma \gtrsim 2m_e$; (II) vacuum turns into plasma
Pulsar electrodynamics (RMM)

- Rotating magnetosphere model (RMM) (assuming highly conducting plasma outside the star)

\[
\vec{E}' = \vec{E} + \frac{\vec{\Omega} \times \vec{r}}{c} \times \vec{B} = 0
\]

i.e., \(E_\parallel = 0\)

- Plasma motion is determined by

\[
\vec{v}_{\text{drift}} = c \frac{\vec{E} \times \vec{B}}{B^2} = \vec{\Omega} \times \vec{r} + j_\parallel \vec{B}
\]

- Corotating plasma is charged

\[
\rho_{\text{GJ}} = \nabla \cdot \vec{E} = -\frac{2}{c} \vec{\Omega} \cdot \vec{B}
\]

Gaps in magnetosphere

• If one assumes that $E_\parallel=0$ everywhere, the magnetic field lines are equipotential ($V = \text{const}$)

• Then,

$$0 = \oint \vec{E} \cdot d\vec{l} = \int (\nabla \times \vec{E}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s}$$

• Thus, $E_\parallel=0$ cannot be enforced everywhere if $\vec{B}$ changes in time

• Regions ("gaps") with unscreened $E_\parallel$ will necessarily develop (they result from dynamical charge/current starvation)

[Ruderman & Sutherland, Astrophys. J. 196, 51 (1975)]
Gaps in magnetosphere

- Gaps can develop at various locations
- Intermittent gaps are caused by rapid outflow of charge
- The gap size $h$ grows at a speed close to the speed of light
- Electric potential difference grows like $\Delta V = E_\parallel h \propto h^2$
- $\Delta V$ & photon flux cause an avalanche production of electron-positron pairs
- Since $B \propto 1/r^3$, anomalous effects are strongest near polar caps

[Ruderman & Sutherland, Astrophys. J. 196, 51 (1975)]
Pulsar gaps

- Estimate of the gap size and the electric field

\[ E_{||} \approx B h / R_{LC} \]

where \( R_{LC} = c / \Omega \) is the radius of light cylinder and

\[ h \simeq 3.6 \text{ m} \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{-3/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{-4/7} \]

The field scales with pulsar parameters as follows

\[ E_{||} \approx 2.7 \times 10^{-8} E_c \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{3/7} \]

where \( E_c = m_e^2 / e = 1.3 \times 10^{18} \text{ V/m} \).

[Ruderman & Sutherland, Astrophys. J. 196, 51 (1975)]
Gap parameters

- Quantitative estimate of the gap size and fields

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where

$$E_c = \frac{m_e^2}{e} = 1.3 \times 10^{18} \text{ V/m}$$

$$B_c = \frac{m_e^2}{e} = 4.4 \times 10^{13} \text{ G}$$
Chiral charge production

• The evolution of the chiral charge is determined by
  \[
  \frac{\partial n_5}{\partial t} + \nabla \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_m n_5
  \]

• While the chiral anomaly produces $n_5$, the chirality flipping tries to wash it away

• The chiral charge $n_5$ approaches the following steady-state value:
  \[
  n_5 = \frac{e^2}{2\pi^2 \Gamma_m} \vec{E} \cdot \vec{B}
  \]

• The estimates for the chirality flip rate in a hot plasma
  \[
  \Gamma_m \approx \frac{\alpha^2 m_e^2}{T} \quad (T \lesssim m_e/\sqrt{\alpha}) \quad \text{and} \quad \Gamma_m \approx \frac{\alpha m_e^2}{T} \quad (T \gg m_e/\sqrt{\alpha})
  \]

[Boyarsky, Cheianov, Ruchayskiy, Sobol, Phys. Rev. Lett. 126, 021801 (2021)]
Time scales

• The gap formation time
  \[ t_h \sim \frac{\hbar}{c} \sim 10^{-8} \text{ s} \]

• Timescale for chiral charge production
  \[ t^* \sim \frac{1}{\Gamma_m} \sim 10^{-17} \text{ s} \]

• Note that
  \[ t_h \gg t^* \]

• Thus, the chirality production is nearly instantaneous
Estimate for $n_5$ in magnetars

- The estimate for the chiral charge is given by

\[ n_5 \simeq \frac{e^2 E \parallel B}{2\pi^2 \Gamma_m} \simeq 1.5 \times 10^{-5} \text{ MeV}^3 \left( \frac{T}{1 \text{ MeV}} \right) \]

\[ \times \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{10/7} \]

- The corresponding chiral chemical potential is

\[ \mu_5 \simeq \frac{3n_5}{T^2} \simeq 4.6 \times 10^{-5} \text{ MeV} \left( \frac{T}{1 \text{ MeV}} \right)^{-1} \]

\[ \times \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{10/7} \]
Values of $n_5$ and $\mu_5$

- The corresponding numerical values for chiral charge and chiral chemical potential are

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CHIRAL PLASMA INSTABILITY

January 11, 2022
Chiral anomalous magnetospheres of magnetars, S@INT seminar, INT, Seattle

Image credit: European Southern Observatory
Plasma with $\mu_5 \neq 0$

- Nonzero $\mu_5$ and $\vec{B}$ drive the chiral magnetic effect (CME)
  \[ \vec{j} = \frac{e^2 \vec{B}}{2\pi^2} \mu_5 \]
- The effect comes from the spin-polarized LLL ($s=\downarrow$)
  - L-handed states ($p_3 < 0$ & $|E| < \mu_5$) are empty (holes with $p_3 > 0$)
  - R-handed states ($p_3 < 0$ & $E < \mu_5$) are occupied
- However, plasma at $\mu_5 \neq 0$ is unstable
Maxwell equations at $\mu_5 \neq 0$

- The total current (CME + Ohm)
  $$j = \frac{2\alpha}{\pi} \mu_5 B + \sigma E$$

- By substituting $j$ into Ampere’s law
  $$\nabla \times B = j + \frac{\partial E}{\partial t}$$
  and solving for the electric field, one derives
  $$E = \frac{1}{\sigma} \left( \nabla \times B - k_\star B - \frac{\partial E}{\partial t} \right)$$
  where $k_\star = \frac{2\alpha \mu_5}{\pi}$

- Finally, by calculating the curl and using Faraday’s law,
  $$\frac{\partial B}{\partial t} = -\frac{1}{\sigma} \left( \nabla \times (\nabla \times B) - k_\star \nabla \times B + \frac{\partial^2 B}{\partial t^2} \right)$$
Helical modes at $\mu_5 \neq 0$

- Search for a solution as a superposition of helical eigenstates

$$\nabla \times \mathbf{B}_{\lambda,k} = \lambda k \mathbf{B}_{\lambda,k}$$

e.g.,

$$\mathbf{B}_{\lambda,k} = B_0 (\hat{x} + i\lambda \hat{y}) e^{-i\omega t + ikz}$$

Then, for a fixed eigenmode, the evolution equation reads

$$\frac{d\mathbf{B}_{\lambda,k}}{dt} = \frac{1}{\sigma} \left( \lambda k_x k - k^2 - \frac{\partial^2}{\partial t^2} \right) \mathbf{B}_{\lambda,k}$$

- The two solutions for the frequency are

$$\omega_{1,2} = -\frac{i}{2} \left( \sigma \pm \sqrt{\sigma^2 + 4k(\lambda k_x - k)} \right)$$
Long-wavelength modes

• For a plasma with high conductivity

\[ \omega_{1,2} \sim \begin{cases} \frac{-i\left(\sigma + \frac{k(\lambda k_* - k)}{\sigma}\right)}{i\frac{k(\lambda k_* - k)}{\sigma}} \end{cases} \]

• The 1\(^{st}\) mode is damped by charge screening:

\[ B_{k,1} \propto B_0 e^{-\sigma t} \]

• The 2\(^{nd}\) mode is unstable when \( k < \lambda k_* \):

\[ B_{k,2} \propto B_0 e^{+tk(\lambda k_* - k)/\sigma} \]

• The momentum of the fastest growing mode \( B_{k,2} \) is

\[ \frac{1}{2}k_* \]
Instability in pulsars

• The estimate for $k_*$

$$k_* \approx 2.2 \times 10^{-7} \text{ MeV} \left( \frac{T}{1 \text{ MeV}} \right)^{-1} \left( \frac{R}{10 \text{ km}} \right)^{2/7} \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{10/7}$$

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Observational consequences

• Unstable plasma in the gaps produces helical (circularly polarized) modes in the frequency range

\[ 0 \leq \omega \leq k_* \]

• For magnetars, these span radio frequencies and may reach into the near-infrared range

• Available energy is of the order of \( \Delta \mathcal{E} \sim \mu_5^2 T^2 h^3 \), i.e.,

\[
\Delta \mathcal{E} \approx 2.1 \times 10^{25} \text{ erg} \left( \frac{T}{1 \text{ MeV}} \right) \left( \frac{R}{10 \text{ km}} \right)^{6/7} \\
\times \left( \frac{\Omega}{1 \text{ s}^{-1}} \right)^{-9/7} \left( \frac{B}{10^{14} \text{ G}} \right)^{2/7}
\]

• The energy is sufficient to feed the fast radio bursts (FRB)
Outstanding problems

• Interplay of chiral charge and electron-positron pair production induced by energetic photons should be studied in detail

• The modification of the chiral flip rate $\Gamma_m \simeq \frac{\alpha^2 m_e^2}{T}$ by the strong magnetic field (extra suppression?)

• The role of the inverse magnetic cascade and the chiral-magnetic turbulence should be quantified

• Self-consistent dynamics of chiral plasma in the gap regions should be simulated in detail

• Detailed mechanism of the energy transfer from unstable helical modes to radio emission in FRBs
Summary

• Chiral anomaly can have *macroscopic* implications in pulsars

• It leads to a *significant* chiral charge production (up to $10^{34} \text{ m}^{-3}$) in strongly magnetized magnetospheres

• The chiral chemical potential $\mu_5$ can be up to $10^{-3} \text{ MeV}$

• This is sufficient to trigger emission of helical waves with frequencies up to about $k_\ast \approx \frac{2}{\pi} \alpha \mu_5$ (radio to infrared range)

• Helical waves can affect the pulsar jets and observable features of the fast radio bursts

• For quantitative effects, further detailed studies are needed