Anomalous chiral matter
and all that
Igor Shovkovy

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Chirality

• Only \textit{massless} Dirac fermions have a well-defined chirality ($\gamma^5 \psi = \pm \psi$):

  \textbf{Right-handed} (spin parallel to momentum)

  \textbf{Left-handed} (spin opposite to momentum)

• \textit{Massive} Dirac fermions have an \textit{almost} well-defined chirality in the \textit{ultrarelativistic} regime*

  – High temperature: $T \gg m$
  – High density: $\mu \gg m$

*Chirality flip rate is nonzero: $\Gamma_{\text{flip}} \propto \alpha^2 T (m/T)^2$
Examples of chiral matter

- **Heavy-ion collisions** (high temperature)

- **Super-dense matter in compact stars** (high density)

- **Early Universe** (high temperature)

- **Magnetospheres of magnetars**
  (electron-positron plasma at moderately high temperature)

- **Electron plasma in Dirac/Weyl (semi-)metals**

- **Other: cold atoms, superfluid $^3$He-A, etc.**
  [Volovik, JETP Lett. 105, 34 (2017)]
DIRAC & WEYL SEMIMETALS


Dirac/Weyl fermions

- Electron quasiparticles with a wide range of properties are possible

- They may even have the emergent spinor structure of **massless** Weyl fermions,

\[ H_W \approx \pm v_F (\vec{\sigma} \cdot \vec{k}) \]

Such nodes are not uncommon!

Na\(_3\)Bi, Cd\(_3\)As\(_2\), ZrTe\(_5\), TaAs, NbAs, …

[Liu et al., Science 343, 864 (2014)]
[Neupane et al., Nature Commun. 5, 3786 (2014)]
[Li et al., Nature Physics 12, 550 (2016)]
[S.-Y. Xu et al., Science 349, 613 (2015)]
[S.-Y. Xu et al., Nature Physics 11, 748 (2015)]
[S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]

Relativistic-like band crossing

A generic 2-band Hamiltonian reads

\[ H_\mathbf{k} = a_\mathbf{k} + \mathbf{b}_\mathbf{k} \cdot \mathbf{\sigma} \quad \Rightarrow \quad E_\mathbf{k} = a_\mathbf{k} \pm \sqrt{(b_\mathbf{k})^2} \]

The bands cross when \[ \mathbf{b}_\mathbf{k} = 0 \]

These 3 equations can be solved by adjusting \( \mathbf{k} \) in 3D
Emergent chirality in solids

Near a band crossing (e.g., $\mathbf{k} \approx \mathbf{k}_+$)

$$H_k = a_{k_+} + (\nabla_k a_k \cdot \delta \mathbf{k}) + \sum_{i,j} \sigma_i b_{ij} \delta k_j$$

After an orthogonal transformation

$$b_{ij} \equiv \partial b_i / \partial k_j \rightarrow \hbar v_i \delta_{ij}$$

Assuming isotropy & a suitable reference point,

$$H_k = \pm v_F (\mathbf{\sigma} \cdot \mathbf{k})$$

which is the Weyl Hamiltonian

The chirality is defined by

$$\lambda = \text{sign} [\det (b_{ij})]$$
Weyl quasiparticles

- The quasiparticle eigenstates for Weyl Hamiltonian $H_\lambda = \lambda v_F (\vec{k} \cdot \vec{\sigma})$ are
  \[
  \psi_k^\lambda = \frac{1}{\sqrt{2/\sqrt{\epsilon_k^2 + \lambda v_F \epsilon_k k_z}}} \left( \begin{array}{c} v_F k_z + \lambda \epsilon_k \\ v_F k_x + i v_F k_y \end{array} \right)
  \]

- The quasiparticle energy $\epsilon_k = v_F \sqrt{k_x^2 + k_y^2 + k_z^2}$ is relativistic-like

- Mapping $k \to \psi_k^\lambda$ has a nontrivial topology

- Consider adiabatic evolution of the wave function from $\psi_k$ to $\psi_{k+\delta k}$:
  \[
  \langle \psi_k | \psi_{k+\delta k} \rangle \approx 1 + \delta \vec{k} \cdot \langle \psi_k | \nabla_k | \psi_k \rangle \approx e^{i a_k \cdot \delta k}
  \]

where $a_k = -i \langle \psi_k | \nabla_k | \psi_k \rangle$ is the Berry connection
Berry curvature & topology

• For Weyl eigenstates, the Berry curvature is

$$\Omega_k \equiv \nabla_k \times a_k = \lambda \frac{\vec{k}}{2k^3}$$

• The Chern number (topological charge)

$$C = \frac{1}{2\pi} \oint \Omega_k \cdot dS_k = \frac{\lambda}{2\pi} \oint \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin \theta \, d\theta d\phi = \lambda$$

• In a solid state material, the Brillouin zone is compact

• A closed surface around a node at $\vec{k}_0$ is also a closed surface around the rest of the Brillouin zone

• Thus, Weyl fermions come in pairs of opposite chirality


[Morimoto & Nagaosa, Scientific Reports 6, 19853 (2016)]
Idealized Dirac and Weyl model

• Low-energy Hamiltonians of a Dirac and Weyl materials

\[
H = \int d^3 r \, \bar{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{p}) - (\vec{b} \cdot \vec{\gamma}) \gamma^5 + b_0 \gamma^0 \gamma^5 \right] \psi
\]

Dirac (e.g., Na\textsubscript{3}Bi, Cd\textsubscript{3}As\textsubscript{2}, ZrTe\textsubscript{5})

Weyl (e.g., TaAs, NbAs, TaP, NbP, WTe\textsubscript{2})
ANOMALOUS EFFECTS

Anomalous chiral matter

• Relativistic matter made of chiral fermions may allow $n_L \neq n_R$ to persist on macroscopic time/distance scales

• The (collective) dynamics of $n_R + n_L$ and $n_R - n_L$ is controlled by the continuity equations

$$\frac{\partial (n_R + n_L)}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$\frac{\partial (n_R - n_L)}{\partial t} + \nabla \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c} - \Gamma_{\text{flip}}(n_R - n_L)$$

**Question:** Can chiral anomaly produce any macroscopic effects in ultra-relativistic matter?
Anomalous effects

• **Theory:** Many *macroscopic* chiral anomalous effects were proposed

• Some are triggered by an external magnetic field
  – Chiral magnetic effect
  – Chiral separation effect
  – Chiral magnetic wave
  – Negative magnetoresistance
  – ...

• Others are triggered by vorticity
  – Chiral vortical effect
  – Chiral vortical wave
  – ...

\[
\langle \vec{j}_5 \rangle = -\frac{e \vec{B}}{2\pi^2} \mu
\]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]
Landau levels

• Landau energy levels at $m = 0$

\[ E_n^{\pm} = \pm \sqrt{2n|eB| + p_z^2} \]

where $n = k + \frac{1}{2} + \text{sgn}(eB)s_z$

where $n = k + \frac{1}{2} + \text{sgn}(eB)s_z$

orbital spin

• Lowest Landau level is spin polarized

\[ E_0^{\pm} = \pm p_z \quad (k = 0, \ s_z = -\frac{1}{2}) \]

• Density of states at $E=0$:

\[ \left. \frac{dn}{dE} \right|_{E=0} = \frac{|eB|}{4\pi^2} \]

• Higher Landau levels ($n \geq 1$) are twice as degenerate:

(i) $k = n \quad \& \quad s = -\frac{1}{2}$

(ii) $k = n - 1 \quad \& \quad s = +\frac{1}{2}$

CSE: Landau spectrum & $\mu \neq 0$

The figure shows the Landau level spectrum $E_n(p_3)$ as a function of $p_3$. The lowest Landau level is partially filled. Right-handed particles are represented by arrows pointing upwards, while left-handed particles are represented by arrows pointing downwards. The chemical potential $\mu$ is indicated by dashed lines.
CSE: Partially filled LLL

- Spin polarized LLL is chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are **R-handed**
  - states with $p_3>0$ (and $s=\downarrow$) are **L-handed**

i.e., a nonzero **axial** current is induced

$$\langle j_5 \rangle = -tr[\gamma \gamma^5 S(x, x)] = -\frac{e\vec{B}}{2\pi^2} \mu$$
\[ \langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5 \]

Chiral Magnetic Effect ($\mu_5 \neq 0$)

Assume that one created a transient state with a nonzero chiral charge ($\mu_5 \neq 0$)

Spin polarized LLL ($s=\downarrow$ for particles of a negative charge):

- Some **R-handed** states ($p_3 < 0 \& E < \mu_5$) are occupied
- Some **L-handed** states ($p_3 < 0 \& |E| < \mu_5$) are empty (i.e., holes with $p_3 > 0$)

CME current: $\langle \vec{j} \rangle = -tr [\vec{\gamma} S(x, x)] = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]
CME: Partially filled LLL ($\mu_5 \neq 0$)

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3 < 0$ (and $s=\downarrow$) are **R-handed particles**
  - states with $p_3 > 0$ (and $s=\downarrow$) are **R-handed antiparticles** (L-handed holes)

i.e., a nonzero **electric current** is induced

$$\langle j \rangle = -tr [\gamma S(x, x)] = \frac{e^2 B}{2\pi^2} \mu_5$$
HEAVY-ION COLLISIONS
$\vec{B}$ and $\vec{\omega}$ in little Bangs

- Rotating & magnetized QGP created at RHIC/LHC
- Electromagnetic fields (Lienard-Wiechert potentials)

- Magnetic field estimate:
  \[ B \sim 10^{18} \text{ to } 10^{19} \text{ G (} \sim 100 \text{ MeV) } \]

- Vorticity estimate: \[ \omega \sim 9 \times 10^{21} \text{ s}^{-1} (\sim 10 \text{ MeV}) \]

[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak & Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108], …
Source of chirality in QCD

- Chiral charge can be produced by topological configurations in QCD

\[
\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3 x \, F_{a\mu\nu} \tilde{F}_{a\mu\nu}^\dagger
\]

- A random fluctuation with nonzero chirality could result in

\[
N_R - N_L \neq 0 \Rightarrow \mu_5 \neq 0
\]

- This should lead to an electric current

\[
\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5
\]
Dipole CME

- Dipole pattern of *charged particle correlations* in heavy-ion collisions

\[
\langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\mp - 2\Psi_{RP}) \rangle > 0 \quad \& \quad \langle \cos(\varphi_\alpha^\pm + \varphi_\beta^\pm - 2\Psi_{RP}) \rangle < 0
\]

[Fukushima, Kharzeev, Waringa, Phys. Rev. D 78, 074033 (2008)]
Correlations of same & opposite charge particles:

\[
\langle \cos(\phi^+_{\alpha} + \phi^-_{\beta} - 2\Psi_{RP}) \rangle > 0
\]

\[
\langle \cos(\phi^+_{\alpha} + \phi^-_{\beta} - 2\Psi_{RP}) \rangle < 0
\]

Large background effects!

[Belmont & Nagle, PRC 96, 024901 (2017)]
CHIRAL MAGNETIC WAVE

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
Chiral Magnetic Wave

- Nonzero charge density @ $B \neq 0 \rightarrow$ CMW

$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2} \mu$$

- Back-to-back electric currents, or quadrupole charge correlations (i.e., difference in elliptic flows of $\pi^+$ and $\pi^-$)

$$\frac{dN_{\pm}}{d\phi} \approx \tilde{N}_\pm [1 + 2\nu_2 \cos(2\phi) \mp A_\pm r \cos(2\phi)]$$

where $A_\pm$ is the charge asymmetry


[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
Dispersion of CMW ($\mathbf{k} \parallel \mathbf{B}$)

- Simple model ($\delta n, \delta n_5 \sim e^{-i k_0 t + i k z}$):

$$k_0 \delta n - \frac{eB}{2\pi^2 \chi_5} k \delta n_5 = 0$$

$$k_0 \delta n_5 - \frac{eB}{2\pi^2 \chi} k \delta n = 0$$

where $\chi_5 \simeq \chi = \partial n / \partial \mu \simeq T^2 / 3$

- The linear dispersion of the CMW mode:

$$k_0 \simeq \pm \frac{eB}{2\pi^2 \chi} k$$

- This is a gapless mode with speed $v \propto eB / T^2$
Background effects may dominate over the signal!

Theory: the chiral magnetic wave might be overlapped...

[Adamczyk et al. (STAR), Phys. Rev. Lett. 114, 252302 (2015)]
[Adam et al. (ALICE), Phys. Rev. C 93, 044903 (2016)]

CMW: careful analysis

- Simple 1-flavor model ($\mathbf{k} \parallel \mathbf{B}$):

\[
k_0 \delta n - k B \delta \sigma_B + i \frac{\tau}{3} k^2 \delta n - \frac{1}{e} \sigma_E k \delta E_z = 0
\]

\[
k_0 \delta n_5 - k B \delta \sigma_B^5 + i \frac{\tau}{3} k^2 \delta n_5 - i \frac{e^2}{2 \pi^2} B \delta E_z = 0
\]

\[k \delta E_z + i e \delta n = 0\]

- The dispersion of the CMW mode:

\[
k_0^{(\pm)} = -i \frac{\sigma_E}{2} \pm i \frac{\sigma_E}{2} \sqrt{1 - \left( \frac{3eB}{\pi^2 T^2 \sigma_E} \right)^2 \left( k^2 + \frac{e^2 T^2}{3} \right) - i \frac{\tau}{3} k^2}
\]

- This is a diffusive mode ($\propto e^{-ik_0 t}$) when

\[
\frac{3eB}{\pi^2 T^2 \sigma_E} \sqrt{k^2 + \frac{e^2 T^2}{3}} < 1
\]
CMW in QCD

Two sets of modes $k_{0,n}^{(\pm)} = \pm E_n(k) - i\Gamma_n(k)$

CMW is completely diffusive at small $eB$ & $k$:
Fresh ideas

Find “knobs” to control/measure magnetic field $\vec{B}$?

• Exploit the beam-energy dependence of the field (⁄)

• Try to measure $\vec{B}$ more directly (⁄)
  – thermal photons (⁄)
  – dilepton rates (⁄)


[Wang, Shovkovy, Yu, Huang, Phys. Rev. D 102, 076010 (2020)]
Isobar collisions

Utilize collisions of isobars, e.g.,

$^{96}_{40}\text{Zr} + ^{96}_{40}\text{Zr} \text{ vs. } ^{96}_{44}\text{Ru} + ^{96}_{44}\text{Ru}$

$^{130}_{52}\text{Te} + ^{130}_{52}\text{Te} \text{ vs. } ^{130}_{56}\text{Ba} + ^{130}_{56}\text{Ba}$

[Voloshin, PRL 105, 172301 (2010)]
[Deng et al. PRC 94, 041901(R) (2016)]

Isobar collisions (theory)

Isobar collisions (experiment)

- Isobar run was completed by STAR in May 2018
- $\approx 3.8$ billion collisions of $^{96}\text{Ru}+^{96}\text{Ru}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ at $\sqrt{s} = 200$ GeV
- Blind analysis by five groups of the STAR Collaboration
- Report announced on Aug. 31, 2021 (online event @ BNL)
- Preprint posted on the same day

Isobar collisions (results)

• Compilation of post-blinding results  [STAR Collaboration, arXiv:2109.00131]

- There is hope for CME… Stay tuned…

- Note two extra data points (open markers):
  - the ratio of inverse multiplicities
  - the ratio of relative pair multiplicity difference
Summary

- Chiral anomaly may have macroscopic implications
- Anomaly may have observable signatures in quark-gluon plasma (and other forms of relativistic matter)
- (Dipole) chiral magnetic effect (CME) can be seen via charged particle correlations in heavy-ion collisions
- Latest isobar measurements are not conclusive yet (active studies are underway)
- Chiral magnetic wave is another phenomenon that may affect quark-gluon plasma (if not overdamped)
- Signatures of CME are tested/observed in Dirac and Weyl semimetals (and other physical systems too)