Photon emission from strongly magnetized QGP

Igor Shovkovy
Arizona State University

[X. Wang, I. Shovkovy, L. Yu, M. Huang, Phys. Rev. D 102, 076010 (2020)]
[X. Wang, I. Shovkovy, in preparation]
PHOTONS AS A THERMOMETER OF QGP

Photons in heavy-ion collisions

- Photons are emitted at all stages of evolution
 Photon sources in HIC


- $p_T \lesssim 2$ GeV: thermal emission dominates
- $2$ GeV $\lesssim p_T \lesssim 4$ GeV: the jet-plasma contribution dominates
Thermal photons (1)

- The rate of the thermal emission of photons (more precisely, the energy loss rate) is

\[
k^0 \frac{d^3 R}{dk_x dk_y dk_z} = - \frac{1}{(2\pi)^3} \frac{\text{Im} \left[ \Pi_{\mu}^\nu(k) \right]}{\exp \left( \frac{k_0}{T} \right) - 1}
\]


- In the case of hot QCD plasma,

![Diagram](image)

- Processes:

![Diagram](image)
Thermal photons (2)

• The approximate result is given by

\[ E \frac{dR}{d^3p} = \frac{5}{9} \frac{\alpha \alpha_s}{2\pi^2} T^2 e^{-E/T} \ln \left( \frac{2.912 E}{g^2 T} \right) \]


• There are important corrections from bremsstrahlung and inelastic pair annihilation

• Next to leading order corrections are \( \sim 100\% \)


[Arnold, Moore, Yaffe, JHEP 05 (2013) 010; arXiv:1302.5970]
Thermal photons (3)

- Numerically,

Quarks and Gluons \( T = 200 \text{ MeV} \)


Photon $v_2$ puzzle

- Most photons are produced early (before flow develops)
- Thus, $v_2$ for photons should be very small

DIRECT PHOTONS AS A MAGNETOMETER OF QGP
Heavy-ion collisions

- QGP produced at RHIC/LHC is magnetized

\[ 10^{18} \text{ to } 10^{19} \text{ G} \sim m_{\pi}^2 \sim (100 \text{ MeV})^2 \]

- Using Lienard-Wiechert potential, one finds

\[
eE(t, \mathbf{x}) = \alpha_{\text{EM}} \sum_{n \in \text{protons}} \frac{1 - v_n^2}{R_n^3 \left(1 - \left[\mathbf{R}_n \times \mathbf{v}_n\right]^2/R_n^2\right)^{3/2}} \mathbf{R}_n
\]

\[
eB(t, \mathbf{x}) = \alpha_{\text{EM}} \sum_{n \in \text{protons}} \frac{1 - v_n^2}{R_n^3 \left(1 - \left[\mathbf{R}_n \times \mathbf{v}_n\right]^2/R_n^2\right)^{3/2}} \mathbf{v}_n \times \mathbf{R}_n
\]

[Rafelski & Müller, PRL, 36, 517 (1976)]
[Kharzeev et al., arXiv:0711.0950]
[Skokov et al., arXiv:0907.1396]
[Voronyuk et al., arXiv:1103.4239]
[Deng & Huang, arXiv:1201.5108]
[Bloczynski et al, arXiv:1209.6594]
...
Magnetic field in HIC

- Magnetic field
  - strong in magnitude $\sim m_\pi^2$
  - depends strongly on $b$
  - nonuniform
  - fluctuates from event to event


[Deng & Huang, Phys. Rev. C 85, 044907 (2012)]
Time dependence

- **Magnetic field**
  - not always $\perp$ to reaction plane
  - short-lived ($\ll 1$ fm/c)
  - conductivity may help a little

Photons from magnetized plasma

- At $\vec{B} \neq 0$, the leading-order polarization tensor leads to a nonzero result!

- All three processes (without the gluon mediation), i.e.,

are allowed by the energy conservation
Photon thermal rate

- The expression for the rate is

\[
\kappa^0 \frac{d^3 R}{d\kappa_x d\kappa_y d\kappa_z} = -\frac{1}{(2\pi)^3} \frac{\text{Im} \left[ \Pi^\mu_{\lambda}(\kappa) \right]}{\exp \left( \frac{k_0}{T} \right) - 1}
\]

At \( \vec{B} \neq 0 \), the imaginary part is

\[
\text{Im} \left[ \Pi^\mu_{R,\lambda}(\Omega; \kappa) \right] = \sum_{f=u,d} \frac{N_c \alpha_f}{2l_f^3} \sum_{n,n'=0}^{\infty} \frac{dp_z}{2\pi} \sum_{\lambda,\eta=\pm1} \frac{n_F(E_{n,p_z,f}) - n_F(\lambda E_{n',p_z-k_z,f})}{2\eta \lambda E_{n,p_z,f} E_{n',p_z-k_z,f}} \sum_{i=1}^{4} F_i^f \delta \left( E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta\Omega \right).
\]

where the Landau level energies are

\[
E_{n,p_z,f} = \sqrt{m^2 + p_{z}^2 + 2n|e_f B|}
\]

Photon thermal rate

- After integrating over $p_z$, the final expression reads

$$\text{Im} \left[ \Pi_{R, \mu}^{\mu} \right] = \sum_{f=u,d} \frac{N_c \alpha_f}{2 \pi l_f^4} \sum_{n>n'} \sum_{n'=0}^\infty \frac{g(n, n') \theta(k_-^f - |k_y|) - \theta(|k_y| - k_+^f)}{\sqrt{[(k_-^f)^2 - k_y^2][(k_+^f)^2 - k_y^2]}} \left( \mathcal{F}_1^f + \mathcal{F}_4^f \right)$$

$$- \sum_{f=u,d} \frac{N_c \alpha_f}{4 \pi l_f^4} \sum_{n=0}^\infty \frac{g_0(n) \theta(|k_y| - k_+^f)}{\sqrt{k_y^2[k_y^2 - (k_+^f)^2]}} \left( \mathcal{F}_1^f + \mathcal{F}_4^f \right),$$

where $g(n, n')$ and $g_0(n)$ are combinations of the Fermi-Dirac distribution functions.

The momentum thresholds are determined by

$$k_\pm^f = \left| \sqrt{m^2 + 2n|e_f B|} \pm \sqrt{m^2 + 2n'|e_f B|} \right|$$

Physics processes

• Real solutions to the energy conservation equation

\[ E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta \Omega = 0 \]

can be found under the following conditions:

\[ q \rightarrow q + \gamma \ (\lambda = +1, \ \eta = -1) : \ \sqrt{\Omega^2 - k_{z-}^2} \leq k_{z-}^f \ \text{and} \ n > n', \]
\[ \bar{q} \rightarrow \bar{q} + \gamma \ (\lambda = +1, \ \eta = +1) : \ \sqrt{\Omega^2 - k_{z-}^2} \leq k_{z-}^f \ \text{and} \ n < n', \]
\[ q + \bar{q} \rightarrow \gamma \ (\lambda = -1, \ \eta = -1) : \ \sqrt{\Omega^2 - k_{z+}^2} \geq k_{z+}^f, \]
Angular dependence: small $k_T$

- Non-smooth dependence on $\phi$ (due to many thresholds)
  Parametrization: $k_x = 0$, $k_y = k_T \cos \phi$ and $k_z = k_T \sin \phi$

- Average rate is maximal at $\phi = \frac{\pi}{2}$ (i.e., $\perp$ to the reaction plane)
Angular dependence: large $k_T$

- Rate quickly decreases with $k_T$
- Average rate is maximal at $\phi = 0$ (i.e., $\parallel$ to the reaction plane)
Nonzero elliptic “flow” ($v_2$)
Thermal rate at $\vec{B} \neq 0$

- The photon production rate
  - decreases with energy ($k_T$) at large $k_T$
  - increases with temperature
  - goes to zero when $k_T \to 0$ (quantization effects)
  - and, thus, has a peak at small nonzero $k_T$

- The thermal rate at $\vec{B} \neq 0$ is relatively large
Quantization @ small $k_T$

- Quantization is important when $k_T \lesssim \sqrt{|eB|}$
  
  - Transitions are possible only at large $p_z$

  $$|p_z| \sim |e_f B| / [k_T(1 + |\sin \phi|)]$$

  - This explains why $\text{Im}(\Pi_\mu^\mu) \to 0$ when $k_T \to 0$

  - Dependence on $\phi$ also explains the negative $v_2$!
Anisotropy of photon emission

- The total rate is

\[
\frac{k^0 d^3 R}{dk_x dk_y dk_z} = R_{1\rightarrow2} + R_{2\rightarrow2} + R_{2\rightarrow3} + \cdots
\]

only at \( \vec{B} \neq 0 \)

even at \( \vec{B} = 0 \)

- \( R_{2\rightarrow1} \):

\( n > n' \)

\( q \)

(\( n, p_z \))

\( q \)

(\( n', p_z - k_z \))

\( \gamma \)

\( k \)

\( n < n' \)

\( \bar{q} \)

(\( n', p_z - k_z \))

\( \gamma \)

\( k \)

\( \gamma \)

\( n', p_z \)

\( \bar{q} \)

\( q \)

\( \gamma \)

\( k \)

\( q \)

\( \bar{q} \)

\( \gamma \)

\( q \)

\( \bar{q} \)

\( \gamma \)

\( q \)

\( \bar{q} \)

\( \gamma \)

\( n, p_z \)

\( n, p_z \)

\( n, p_z \)

\( n, p_z \)
Magnetic enhancement of $v_2$

- Estimate of $v_2$ in a hot magnetized QGP

\[ \mathcal{R}_{2\to1} : \quad v_2 \sim 20\% \]

- Noting that

\[ \mathcal{R}_{2\to1} \geq \mathcal{R}_{2\to2} \geq \mathcal{R}_{2\to3} \]

- Naïve estimate at $p_T \sim 1$ GeV gives

\[ 6.7\% \leq v_2 \leq 20\% \]

- A more realistic estimate should consider non-isotropic expansion & non-thermal processes
Summary

• At $\vec{B} \neq 0$, photons are produced at 0$^{th}$ order in $\alpha_s$

$$\begin{align*}
\text{(i) } q &\rightarrow q + \gamma, \\
\text{(ii) } \bar{q} &\rightarrow \bar{q} + \gamma, \\
\text{(iii) } q + \bar{q} &\rightarrow \gamma
\end{align*}$$

• The annihilation contribution grows with $k_T$

• Quantization effects are important for $k_T \lesssim \sqrt{|eB|}$

• Photon emission has pronounced ellipticity

  \[ v_2 < 0 \text{ at small } k_T \left( k_T \lesssim \sqrt{|eB|} \right) \]

  \[ v_2 > 0 \text{ at large } k_T \left( k_T \gtrsim \sqrt{|eB|} \right) \]

• Nonzero ellipticity of thermal emission could be used to "measure" the magnetic field