Direct photons from magnetized quark-gluon plasma

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[X. Wang, I. Shovkovy, in preparation]
MAGNETIZED RELATIVISTIC PLASMA
Magnetized plasmas

• Early Universe

\[ 10^{20} \text{ to } 10^{24} \text{ G } \sim (1 \text{ GeV})^2 \text{ to } (100 \text{ GeV})^2 \]

• Heavy-ion collisions (this talk)

\[ 10^{18} \text{ to } 10^{19} \text{ G } \sim (100 \text{ MeV})^2 \]

• Super-dense matter in magnetars

\[ 10^{14} \text{ to } 10^{16} \text{ G } \sim (1 \text{ MeV})^2 \text{ to } (10 \text{ MeV})^2 \]

• Electrons in Dirac/Weyl (semi-)metals

\[ \lesssim 10^5 \text{ G } \sim (100 \text{ meV})^2 \]
• QGP produced at RHIC/LHC is magnetized

– $10^{18}$ to $10^{19}$ G $\sim m_\pi^2 \sim (100 \text{ MeV})^2$

• Using Lienard-Wiechert potential, one finds

\[
eE(t, \mathbf{x}) = \alpha_{\text{EM}} \sum_{n \in \text{protons}} \frac{1 - v_n^2}{R_n^3 (1 - [\mathbf{R}_n \times \mathbf{v}_n]^2/R_n^2)^{3/2}} \mathbf{R}_n
\]

\[
eB(t, \mathbf{x}) = \alpha_{\text{EM}} \sum_{n \in \text{protons}} \frac{1 - v_n^2}{R_n^3 (1 - [\mathbf{R}_n \times \mathbf{v}_n]^2/R_n^2)^{3/2}} \mathbf{v}_n \times \mathbf{R}_n
\]

[Rafelski & Müller, PRL, 36, 517 (1976)]
[Kharzeev et al., arXiv:0711.0950]
[Skokov et al., arXiv:0907.1396]
[Voronyuk et al., arXiv:1103.4239]
[Deng & Huang, arXiv:1201.5108]
[Bloczynski et al, arXiv:1209.6594]
...
Magnetic field in HIC

- Magnetic field
  - strong in magnitude $\sim m_{\pi}^2$
  - depends strongly on $b$
  - nonuniform
  - fluctuates from event to event


[Deng & Huang, Phys. Rev. C 85, 044907 (2012)]

$e \cdot \langle \text{Field} \rangle / m_{\pi}^2$

$Au+Au, \sqrt{s} = 200 \text{ GeV}$

$b (fm)$

$0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$
Time dependence

• Magnetic field
  – not always $\perp$ to reaction plane
  – short-lived ($\ll 1\text{ fm/c}$)
  – conductivity may help a little


\[ eB_y/m_\pi \]

\[ \frac{1}{2} s_{NN} = 200 \text{ GeV} \]

\[ b = 4 \text{ fm} \]

\[ b = 2 \text{ fm} \]

\[ b = 6 \text{ fm} \]

\[ b = 10 \text{ fm} \]


ANOMALOUS EFFECTS IN HEAVY-ION COLLISIONS


https://physics.aps.org/articles/v2/104
CME, CSE, CMW, etc.

- Chiral magnetic/separation effects, chiral magnetic waves

\[ \langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2} \mu_5 \quad \& \quad \langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2} \mu \]

- Signs of local P-violation?

\[
\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_\mu^\nu \tilde{F}_\mu^\nu
\]

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]

- Signs of chiral magnetic wave?

[Yee, Kharzeev, Phys. Rev. D 83, 085007 (2011)]
[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
[Shovkovy, Rybalka, Gorbar, arXiv: 1811.10635]
Correlations of same & opposite charge particles:

\[
\begin{align*}
\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle \\
\langle \cos(\phi_\alpha^\pm + \phi_\beta^\mp - 2\Psi_{RP}) \rangle
\end{align*}
\]

[Adamczyk et al. (STAR), PRC 88, 064911 (2013)]
Background effects may dominate over the signal!

[Adamczyk et al. (STAR), Phys. Rev. Lett. 114, 252302 (2015)]
[Adam et al. (ALICE), Phys. Rev. C 93, 044903 (2016)]

- On the theoretical side: CMW is likely to be overdamped (unless magnetic field is very strong)  [Shovkovy, Rybalka, Gorbar, arXiv:1811.10635]
How to measure $\vec{B}$?

• One of the ideas [STAR Collaboration, 2014]

  – “measure” the relative strengths of the effects in isobar collisions, e.g., [Koch, Schlichting, Skokov et al., arXiv:1608.00982]

    $^{96}_{40}$Zr + $^{96}_{40}$Zr vs. $^{96}_{44}$Ru + $^{96}_{44}$Ru

    $^{130}_{52}$Te + $^{130}_{52}$Te vs. $^{130}_{56}$Ba + $^{130}_{56}$Ba

• Any chance of measuring $\vec{B}$ directly?

• Perhaps an electromagnetic probe?

• Current proposal:

  – thermal photons

  – dilepton rates
PHOTONS AS A THERMOMETER OF QGP

Photons in heavy-ion collisions

- Photons are emitted at all stages of evolution

https://u.osu.edu/vishnu/category/visualization/
• $p_T \lesssim 2$ GeV: thermal emission dominates

• $2$ GeV $\lesssim p_T \lesssim 4$ GeV: the jet-plasma contribution dominates
Thermal photons (1)

• The rate of the thermal emission of photons (more precisely, the energy loss rate) is

\[ k_0 \frac{d^3 R}{dk_x dk_y dk_z} = - \frac{1}{(2\pi)^3} \frac{\text{Im} \left[ \Pi_{\mu}^{\nu} (k) \right]}{\exp \left( \frac{k_0}{T} \right) - 1} \]


• In the case of hot QCD plasma,

[Diagram showing processes]

• Processes:

[Diagram showing processes]
Thermal photons (2)

- The approximate result is given by

\[ E \frac{dR}{d^3p} = \frac{5 \alpha \alpha_s}{9 \pi} T^2 e^{-E/T} \ln \left( \frac{2.912 E}{g^2 T} \right) \]


- There are important corrections from bremsstrahlung and inelastic pair annihilation


- Next to leading order corrections are \( \sim 100\% \)


[Arnold, Moore, Yaffe, JHEP 05 (2013) 010; arXiv:1302.5970]
- Numerically,

\[ \text{Quarks and Gluons } T = 200 \text{ MeV} \]


Photon $v_2$ puzzle

- Most photons are produced early (before flow develops)

- Thus, $v_2$ for photons should be very small

DIRECT PHOTONS AS A MAGNETOMETER OF QGP
Photons from magnetized plasma

- At $\vec{B} \neq 0$, the leading-order polarization tensor leads to a nonzero result!
- All three processes (without the gluon mediation), i.e.,

are allowed by the energy conservation
Photon thermal rate

- The expression for the rate is

\[
k_0^{0} \frac{d^3 R}{dk_x dk_y dk_z} = - \frac{1}{(2\pi)^3} \frac{\text{Im} \left[ \Pi_{\mu}^{\mu}(k) \right]}{\exp \left( \frac{k_0}{T} \right) - 1}
\]

At \( \vec{B} \neq 0 \), the imaginary part is

\[
\text{Im} \left[ \Pi_{R,\mu}^{\mu}(\Omega; k) \right] = \sum_{f=u,d} \frac{N_c \alpha_f}{2 l_f^3} \sum_{n,n'=0}^{\infty} \int \frac{dp_z}{2\pi} \sum_{\lambda,\eta=\pm1} \frac{n_F(E_{n,p_z,f}) - n_F(\lambda E_{n',p_z-k_z,f})}{2\eta\lambda E_{n,p_z,f} E_{n',p_z-k_z,f}} \sum_{i=1}^{4} F_i^f \delta (E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta\Omega)
\]

where the Landau level energies are

\[
E_{n,p_z,f} = \sqrt{m^2 + p_z^2 + 2n|e_f B|}
\]

Photon thermal rate

- After integrating over $p_z$, the final expression reads

$$\text{Im} \left[ \Pi_{R,\mu}^{\mu} \right] = \sum_{f=u,d} \frac{N_c \alpha_f}{2 \pi l_f^4} \sum_{n>n'}^{\infty} g(n, n') \frac{\theta \left( k_{-}^f - |k_y| \right) - \theta \left( |k_y| - k_{+}^f \right)}{\sqrt{[(k_{-}^f)^2 - k_y^2][(k_{+}^f)^2 - k_y^2]}} \left( \mathcal{F}_1^f + \mathcal{F}_4^f \right)$$

$$- \sum_{f=u,d} \frac{N_c \alpha_f}{4 \pi l_f^4} \sum_{n=0}^{\infty} g_0(n) \frac{\theta \left( |k_y| - k_{+}^f \right)}{\sqrt{k_y^2[k_y^2 - (k_{+}^f)^2]}} \left( \mathcal{F}_1^f + \mathcal{F}_4^f \right),$$


where $g(n, n')$ and $g_0(n)$ are combinations of the Fermi-Dirac distribution functions.

The momentum *thresholds* are determined by

$$k_{\pm}^f = \left| \sqrt{m^2 + 2n |e_f B|} \pm \sqrt{m^2 + 2n' |e_f B|} \right|$$
Physics processes

- Real solutions to the energy conservation equation

\[ E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta \Omega = 0 \]

can be found under the following conditions:

\[ q \rightarrow q + \gamma \ (\lambda = +1, \ \eta = -1) : \quad \sqrt{\Omega^2 - k_z^2} \leq k_+^f \text{ and } n > n', \]

\[ \bar{q} \rightarrow \bar{q} + \gamma \ (\lambda = +1, \ \eta = +1) : \quad \sqrt{\Omega^2 - k_z^2} \leq k_-^f \text{ and } n < n', \]

\[ q + \bar{q} \rightarrow \gamma \ (\lambda = -1, \ \eta = -1) : \quad \sqrt{\Omega^2 - k_z^2} \geq k_+^f, \]
Angular dependence: small $k_T$

- Non-smooth dependence on $\phi$ (due to many thresholds)
  Parametrization: $k_x = 0$, $k_y = k_T \cos \phi$ and $k_z = k_T \sin \phi$

- Average rate is maximal at $\phi = \frac{\pi}{2}$ (i.e., $\perp$ to the reaction plane)
Angular dependence: large $k_T$

- Rate quickly decreases with $k_T$
- Average rate is maximal at $\phi = 0$ (i.e., $\parallel$ to the reaction plane)

August 18, 2020
Webinar on Quark Matter, Sharif University of Technology, Tehran
Nonzero elliptic “flow” ($v_2$)
Thermal rate at $\vec{B} \neq 0$

- The photon production rate
  - decreases with energy ($k_T$) at large $k_T$
  - increases with temperature
  - goes to zero when $k_T \to 0$ (quantization effects)
  - and, thus, has a peak at small nonzero $k_T$

- The thermal rate at $\vec{B} \neq 0$ is relatively large

[Graphs showing the thermal rate for different $|eB|$ and temperatures]

[References: Kapusta et al. 1991]
Quantization @ small $k_T$

- Quantization is important when $k_T \lesssim \sqrt{|eB|}$
  
  - Transitions are possible only at large $p_z$
    
    $$|p_z| \sim |e_f B|/ [k_T (1 + |\sin \phi|)]$$
  
  - This explains why $\text{Im}(\Pi_{\mu}^\mu) \to 0$ when $k_T \to 0$
  
  - Dependence on $\phi$ also explains the negative $v_2$!
Anisotropy of photon emission

- The total rate is

\[ \frac{k^0 d^3 R}{dk_x dk_y dk_z} = R_{1\rightarrow 2}^{2\rightarrow 1} + R_{2\rightarrow 2} + R_{2\rightarrow 3}^{3\rightarrow 2} + \cdots \]

only at \( \vec{B} \neq 0 \) even at \( \vec{B} = 0 \)
Magnetic enhancement of $\nu_2$

- Estimate of $\nu_2$ in a hot magnetized QGP
  \[
  \mathcal{R}_{2\rightarrow1}^{1\rightarrow2}: \quad \nu_2 \sim 20\%
  \]

- Noting that
  \[
  \mathcal{R}_{2\rightarrow1}^{1\rightarrow2} \geq \mathcal{R}_{2\rightarrow2}^{1\rightarrow2} \geq \mathcal{R}_{2\rightarrow3}^{3\rightarrow2}
  \]

- Naïve estimate at $p_T \sim 1$ GeV gives
  \[6.7\% \lesssim \nu_2 \lesssim 20\%\]

- A more realistic estimate should consider non-isotropic expansion & non-thermal processes
Summary

• At $\vec{B} \neq 0$, photons are produced at 0th order in $\alpha_s$
  
  (i) $q \rightarrow q + \gamma$,  
  (ii) $\bar{q} \rightarrow \bar{q} + \gamma$,  
  (iii) $q + \bar{q} \rightarrow \gamma$

• The annihilation contribution grows with $k_T$

• Quantization effects are important for $k_T \lesssim \sqrt{|eB|}$

• Photon emission has pronounced ellipticity
  
  $- \nu_2 < 0$ at small $k_T$ ($k_T \lesssim \sqrt{|eB|}$)
  
  $- \nu_2 > 0$ at large $k_T$ ($k_T \gtrsim \sqrt{|eB|}$)

• Nonzero ellipticity of thermal emission could be used to “measure” the magnetic field