Chiral effects in strong magnetic backgrounds: from QCD to condensed matter physics

Igor Shovkovy
Arizona State University
Relativistic *heavy-ion collisions* produce strong magnetic fields

\(10^{18} - 10^{19} \text{ Gauss} \ (\sqrt{|eB|} \sim 100 \text{ MeV})\)

Quark matter may form inside *magnetars*

\(10^{14} - 10^{16} \text{ Gauss} \ (\sqrt{|eB|} \sim 1 \text{ MeV to 10 MeV})\)

Strong magnetic field is an instructive *theoretical tool* to study confined gauge theories such as QCD

\(\geq 10^{19} \text{ Gauss} \ (\sqrt{|eB|} \geq 100 \text{ MeV to 10 MeV})\)
QCD in magnetic field

- QCD is strongly coupled & nonperturbative
- There are theoretical tools that provide insight
  - High-energy (weak-coupling) expansion
  - Large $N_c$ expansion
  - High temperature limit ($T \gg \Lambda_{QCD}$)
  - High density limit ($\mu \gg \Lambda_{QCD}$)
  - Lattice QCD
- Strong magnetic field $B$ is yet another tool
  - it probes physics at short distances $\ell \sim 1/\sqrt{|eB|}$
- Coupling constant in QCD runs with the energy scale,

\[ \frac{1}{\alpha_s(\mu)} \simeq b \ln \frac{\mu^2}{\Lambda_{QCD}^2}, \quad \text{where} \quad b = \frac{11 N_c - 2 N_f}{12\pi} \]

- The question is: What happens in a strong magnetic field?

ArXiv:1410.6765
QCD ground state at $\vec{B} \rightarrow \infty$

- Lagrangian density of QCD in an external magnetic field

$$\mathcal{L} = -\frac{1}{2} F_{\mu\nu}^A F_{\mu\nu}^A + \bar{\psi}_f (i \gamma^\mu D_\mu) \psi_f$$

where $D_\mu = \partial_\mu + igA_\mu^A \lambda^A / 2 + ie_f A_\mu^{ext}$

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + g f^{ABC} A_\mu^B A_\nu^C$$

- The global chiral symmetry of the model

$$SU_L(N_u) \times SU_R(N_u) \times SU_L(N_d) \times SU_R(N_d) \times U_A^(-)(1)$$

- Quark masses $m_u \neq m_d \neq 0$ break the symmetry down to

$$SU_V(N_u) \times SU_V(N_d)$$
QCD at $|eB| \gg \Lambda_{QCD}^2$

- Energy scales in the problem at hand

- Confined gluodynamics, glueballs

- Magnetic catalysis in weakly coupled QCD and strong B-field, strong gluon screening

- Deep-UV region with asymptotic freedom and weak B-field

$$\frac{1}{\alpha_s(\mu)} \approx b_0 \ln \frac{\mu^2}{\Lambda_{QCD}^2}$$

$$\frac{1}{\alpha_s(\mu)} \approx b \ln \frac{\mu^2}{\Lambda_{QCD}^2}$$
Running $\alpha_s$ in QCD at strong B

- In deep UV region, $\alpha_s$ is not affected by B-field.

\[ \frac{1}{\alpha_s} \approx b \ln \frac{|eB|}{\Lambda_{QCD}^2} \]

\[ M_g^2 \approx \frac{\alpha_s}{\pi} \sum_f |e_f B| \]

\[ m_{dyn}^2 \ll |k^2| \ll |eB| \]
The general form of the equation is similar to that in QED

\[ G^{-1}(x, y) = G_0^{-1}(x, y) + 4\pi\alpha_s \gamma^\mu T^A G(x, y) \gamma^\nu T^B D^{AB}_{\mu\nu}(y - x) \]

Note: \( G^{-1}(x, y) \) and \( G(x, y) \) have same Schwinger phases!

Screening effects are included via the polarization function in the strong field limit (\( \sqrt{|eB|} \gg \Lambda_{QCD} \))

\[
\mathcal{P}^{AB,\mu\nu} \sim \frac{\alpha_s}{6\pi} \delta^{AB} \left(k_\parallel k'_\parallel - k_\parallel^2 g_\parallel^{\mu\nu}\right) \sum_{q=1}^{N_f} \frac{|e_q B|}{m_q^2}, \quad \text{for } |k_\parallel^2| \ll m_q^2
\]

\[
\mathcal{P}^{AB,\mu\nu} \sim -\frac{\alpha_s}{\pi} \delta^{AB} \left(k_\parallel k'_\parallel - k_\parallel^2 g_\parallel^{\mu\nu}\right) \sum_{q=1}^{N_f} \frac{|e_q B|}{k_\parallel^2}, \quad \text{for } m_q^2 \ll |k_\parallel^2| \ll |eB|
\]
Screening masses: lattice

- Electric and magnetic screening masses on the lattice grow with the field \[ \text{[Bonati et al., Phys. Rev. D 95, 074515 (2017)]} \]

\[
\frac{m^d_E}{T} = a^d_E \left[ 1 + c^d_{1;E} \frac{|e|B}{T^2} \tan \left( \frac{c^d_{2;E}}{c^d_{1;E}} \frac{|e|B}{T^2} \right) \right]
\]

(and a similar expression for the magnetic one)
Expression for dynamical mass

- In the region $m_{dyn}^2 \ll |k|^2 \ll |eB|$, which is most relevant for the fermion-pairing dynamics, the gluon has a “mass”

$$M_g^2 \simeq \frac{\alpha_s}{\pi} \sum_f |e_f B| = \frac{\alpha_s}{3\pi} (2N_u + N_d) |eB|$$

- **Rigorous** SD analysis (with higher-order diagrams under control) can be performed by using a special non-local gauge for the gluon propagator

- The final result reads,

$$m_q^2 \simeq 2C_1 |e_q B| (c_q \alpha_s)^{2/3} \exp \left[ -\frac{4N_c \pi}{\alpha_s(N_c^2 - 1) \ln(C_2/c_q \alpha_s)} \right]$$

where $C_1 \simeq C_2 \simeq 1$ and $c_q \simeq (2N_u + N_d)|e|/(6\pi |e_q|)$
Quark mass vs. $B$

- Quantitatively, dynamical masses are $(\sqrt{|eB|} \gg \Lambda_{\text{QCD}})$

[Graph showing the relationship between $m_u/m_\Lambda_{\text{QCD}}$, $m_d/m_\Lambda_{\text{QCD}}$, and $\ln(|eB|/\Lambda^2_{\text{QCD}})$ with $\alpha_s \gtrsim 0.1$ indicating a region of interest at $T=0$.]

Chiral condensate in lattice QCD

\[ \Sigma = \mathcal{W} \propto \mathcal{W} \]

\[ \Delta(\Sigma_u + \Sigma_d) / 2 \]

\[ \Sigma_f = \bar{\psi}_f \psi_f \propto m_{\text{dyn},f} \]

\[ T=0 \]

\[ eB \ (\text{GeV}^2) \]

[\text{Bali et al., Phys. Rev. D86, 071502 (2012)}]
Nambu-Goldstone bosons (pions)

• Original global chiral symmetry

\[ SU_L(N_u) \times SU_R(N_u) \times SU_L(N_d) \times SU_R(N_d) \times U_A^- \]  

breaks down to

\[ SU_V(N_u) \times SU_V(N_d) \]

• Thus, there should be \((N_u^2 + N_d^2 - 1)\) massless NG bosons

• The unitary pion fields can be written in terms of the coset space generators

\[ \Sigma_u \equiv \exp \left( i \sum_{A=1}^{N_u^2-1} \lambda^A \pi^A_u / f_u \right), \quad \Sigma_d \equiv \exp \left( i \sum_{A=1}^{N_d^2-1} \lambda^A \pi^A_d / f_d \right) \]

and \[ \tilde{\Sigma} \equiv \exp \left( i \sqrt{2} \tilde{\pi} / \tilde{f} \right) \]

• In a very strong magnetic field another light pseudo-NG boson, associated with anomalous \(U_A(1)\), may appear
Low-energy action for NG bosons

• The low-energy effective action should have the form

\[ \mathcal{L}_{\text{NG}} \simeq \frac{f_u^2}{4} \text{tr} \left( g_{\mu\nu} \partial_\mu \Sigma_u \partial_\nu \Sigma_u^\dagger + v_u^2 g_{\mu\nu} \partial_\mu \Sigma_u \partial_\nu \Sigma_u^\dagger \right) + \ldots \]

• The pion decay constants are defined by

\[ i \left\langle 0 \vbar \gamma^\mu \gamma^5 \frac{\lambda^A}{2} \psi \right| \pi^B (P) \rangle = p^\mu f_\pi \delta^{AB} = -i \int \frac{d^4 k}{(2\pi)^4} \text{tr} \left( \gamma^\mu \gamma^5 \frac{\lambda^A}{2} \chi^B_q (k, P) \right) \]

where \( P^\mu = (P^0, v^2 \bar{P}, P^3) \)

• The Bethe-Salpeter wave function looks like in QED in LLL approximation. So, we find that \( v^2 \approx 0 \), and

\[ f_q^2 = 4N_c \int \frac{d^2 k^\perp d^2 k^\parallel}{(2\pi)^4} \exp \left( -\frac{k^2}{|e_q B|} \right) \frac{m_q^2}{(k^2 + m_q^2)^2} \]

which can be easily calculated, giving

\[ f_u^2 = \frac{N_c |eB|}{6\pi^2} \quad \text{and} \quad f_d^2 = \frac{N_c |eB|}{12\pi^2} \]
Far IR region, $|k_\parallel^2| \leq m_{dyn}^2$

- Massive quarks decouple from the low-energy dynamics

- Gluons are the only “light” degrees of freedom

- Assuming that $\Lambda_{QCD}^2 \ll m_{dyn}^2$, the gluodynamics has a semi-perturbative region, $|k_\parallel^2| \lesssim m_{dyn}^2$, where

$$\frac{1}{\tilde{\alpha}_s(\mu)} - \frac{1}{\alpha_s} \simeq b_0 \ln \frac{\mu^2}{m_{dyn}^2}$$

Here $b_0 = \frac{11 N_c}{12\pi}$ and $\frac{1}{\alpha_s} \simeq b \ln \frac{|eB|}{\Lambda_{QCD}^2}$ (Recall: $b = \frac{11 N_c - 2N_f}{12\pi}$)

- Then, we find that the new confinement scale where $\tilde{\alpha}_s = \infty$:

$$-b \ln \frac{|eB|}{\Lambda_{QCD}^2} \simeq b_0 \ln \frac{\Lambda_{QCD}^2}{m_{dyn}^2} \Rightarrow \lambda_{QCD} = m_{dyn} \left( \frac{\Lambda_{QCD}}{\sqrt{|eB|}} \right)^{b/b_0}$$
• Quadratic part of low-energy effective action for gluons

\[ \mathcal{L}_{\text{glue, eff}}^{(2)} = -\frac{1}{2} \sum_{A=1}^{N_c^2-1} A^A_{\mu}(-k) \left[ g^{\mu\nu} k^2 - k^\mu k^\nu + \kappa \left( g^{\mu\nu} k^2_{||} - k^\mu_{||} k^\nu_{||} \right) \right] A^A_{\nu}(k) \]

where the susceptibility \( \kappa \) is extracted from the polarization tensor \( \mathcal{P}_{\mu\nu}^{AB} \) in the region \( |k_{||}| \ll m_{\text{dyn}}^2 \), i.e.,

\[ \kappa = \frac{\alpha_s}{6\pi} \sum_{q=1}^{N_f} \frac{|e_q B|}{m_q^2} = \frac{1}{12C_1\pi} \sum_{q=1}^{N_f} \left( \frac{\alpha_s}{c_q^2} \right)^{1/3} \exp \left( \frac{4N_c\pi}{\alpha_s(N_c^2 - 1) \ln(C_2/c_q\alpha_s)} \right) \gg 1 \]

• The requirement of gauge invariance allows to write down the complete expression for the gluon action

\[ \mathcal{L}_{\text{glue, eff}} \sim \frac{1}{2} \sum_{A=1}^{N_c^2-1} \left( \mathbf{E}_\perp^A \cdot \mathbf{E}_\perp^A + \epsilon E_3^A E_3^A - \mathbf{B}_\perp^A \cdot \mathbf{B}_\perp^A - B_3^A B_3^A \right) \]

where \( \epsilon = 1 + \kappa \) is a chromo-dielectric constant (note \( \epsilon \gg 1 \)), \( E_i^A = F_{0i}^A \) and \( B_i^A = \frac{1}{2} \varepsilon_{ijk} F_{jk}^A \) are chromo-fields.
By using the guidance from an analogous *anisotropic* QED, the static potential between a pair of quarks should be given by

\[
V(x, y, z) = -\frac{g_s^2}{4\pi\sqrt{z^2 + \epsilon(x^2 + y^2)}}
\]

which is valid for a range of distance scales \( m_{dyn}^{-1} \approx r \approx \lambda_{QCD}^{-1} \)
Anisotropy in detail

- The dependence of the potential as a function of angle $\theta$ between $\vec{B}$ and $q\bar{q}$ orientation [Bonati et al., Phys. Rev. D 94, 094007 (2016)]

$$V(r, \theta; B) = -\frac{\alpha(\theta; B)}{r} + \sigma(\theta; B)r + V_0(\theta; B)$$

- With increasing angle $\theta$, the string tension increases
• What to expect at nonzero temperature (in strong B limit)?

• Very low temperatures, $T \ll \lambda_{QCD}$
  – Ground state in not affected much
  – Color is confined, lowest energy states are glueballs
  – Chiral symmetry is broken ($T \ll \lambda_{QCD} \ll m_{dyn}$)

• Intermediate temperatures, $\lambda_{QCD} \ll T \ll m_{dyn}$
  – Color is deconfined; gluons are thermally populated
  – Chiral symmetry is still broken ($\lambda_{QCD} \ll T \ll m_{dyn}$)

• Moderately high temperatures, $m_{dyn} \ll T \ll \sqrt{|eB|}$
  – Chiral symmetry is restored ($m_{dyn} \ll T$)
Predicted phase diagram

$T_c \sim m_{\text{dyn}}$ (chiral symmetry restoration)

$T_c^* \sim \lambda_{\text{QCD}}$ (deconfinement)

Dependence of $T_c$ vs. $B$

![Graph showing the dependence of $T_c$ vs. $B$](image)

- Strange quark number susceptibility
- Average light quark condensate
- Polyakov loop

Valence vs. sea

Valence

Sea

\[ \Delta \Sigma^\text{val}_u \]

\[ \Delta \Sigma^\text{sea}_u \]

- Gluon screening (?)
- Polyakov loops (?)

or, perhaps, something else (?)

[Bruckmann, Endrodi, Kovacs, JHEP 04 (2013) 112]
Super-strong $B$: prediction

![Graph showing deconfinement transition line, prediction, crossover, critical endpoint, and first order.

Predicted phase diagram

$T_c \sim m_{\text{dyn}}$ (chiral symmetry restoration)

$T_c^* \sim \lambda_{\text{QCD}}$ (deconfinement)

CHIRAL MATTER
Chiral matter

- Matter made of chiral fermions with \( n_L \neq n_R \)

- Unlike the electric charge \((n_R + n_L)\), the chiral charge \((n_R - n_L)\) is **not** conserved

\[
\frac{\partial (n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0
\]

\[
\frac{\partial (n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2 c}
\]

- The chiral symmetry is anomalous in quantum theory
Chiral separation effect ($\mu \neq 0$)

Spin ($s=\downarrow$) polarized LLL:
- $p_3 < 0$ states are R-handed
- $p_3 > 0$ states are L-handed

This leads to CSE:

$$\langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2} \mu$$

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]
Chiral magnetic effect ($\mu_5 \neq 0$)

- Right-handed
- Left-handed

Spin ($s=\downarrow$) polarized LLL:
- $p_3 < 0$ states are R-handed electrons
- $p_3 > 0$ states are L-handed positrons

This leads to CME:

$$\langle \bar{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]
CMW/Quadrupole CME

- Start from a small baryon density and $B \neq 0$

\[
\langle \hat{j}_5 \rangle = \frac{eB}{2\pi^2} \mu
\]

\[
\langle \hat{j} \rangle = \frac{eB}{2\pi^2} \mu_5
\]

- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
Dirac & Weyl materials

• Na$_3$Bi
  [Z. K. Liu et al., Science 343, 864 (2014)]

• Cd$_3$As$_2$
  [M. Neupane et al., Nature Commun. 5, 3786 (2014)]

• ZrTe$_5$
  [X. Li et al., Nature Physics 12, 550 (2016)]

E = $\sqrt{v_x^2 k_x^2 + v_y^2 k_y^2 + v_z^2 k_z^2}$, \(v_x \approx v_y \approx 3.74 \times 10^5\) m/s, \(v_z \approx 2.89 \times 10^4\) m/s

• TaAs (tantalum arsenide)
  [S.-Y. Xu et al., Science 349, 613 (2015)]

• NbAs (niobium arsenide)
  [S.-Y. Xu et al., Nature Physics 11, 748 (2015)]

• TaP (tantalum phosphide)
  [S.-Y. Xu et al., Science Adv. 1, 1501092 (2015)]

• NbP (niobium phosphide)

• WTe$_2$ (tungsten telluride)
Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

\[
H = \int d^3 r \bar{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{p}) - (\vec{b} \cdot \vec{\gamma}) \gamma^5 + b_0 \gamma^0 \gamma^5 \right] \psi
\]

Dirac

Weyl
Strain in Weyl materials

- Strains affect low-energy quasiparticles in Weyl materials

\[
H = \int d^3 r \overline{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{p}) - (\vec{b} + \vec{A}_5) \cdot \vec{\gamma} \gamma^5 + (b_0 + A_{5,0}) \gamma^0 \gamma^5 \right] \psi
\]

where the components of the chiral gauge fields are

\[
\begin{align*}
A_{5,0} &\propto b_0 |\vec{b}| \partial_|| u_|| \\
A_{5,\perp} &\propto |\vec{b}| \partial_|| u_\perp \\
A_{5,||} &\propto \alpha |\vec{b}|^2 \partial_|| u_|| + \beta \sum_i \partial_i u_i
\end{align*}
\]

The associated pseudo-EM fields are

\[
\begin{align*}
\vec{B}_5 &= \vec{\nabla} \times \vec{A}_5 \\
\vec{E}_5 &= -\vec{\nabla} A_0 - \partial_t \vec{A}_5
\end{align*}
\]
Chiral effects

• Any signature properties of Dirac/Weyl materials directly sensitive to chiral anomaly?

• Some proposals:
  – Anomalous Hall effect
  – Anomalous Alfven waves
  – Strain/torsion induced CME
  – Strain/torsion induced quantum oscillations
  – Strain/torsion dependent resistance
  – etc.

• Spectrum of chiral (pseudo-)magnetic plasmons

  [Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 95, 115202 (2017)]
General question

• What are the properties of plasmons in magnetized chiral material with \( b_0 \neq 0 \) and \( \vec{b} \neq 0 \)?

• Chiral matter \((\mu_R \neq \mu_L)\)
  
  – This is the case in equilibrium when \( b_0 \neq 0 \) \((\mu_5 = -eb_0)\)

• Magnetic or pseudomagnetic field is present

In general, \( E_\lambda = E + \lambda E_5 \) and \( B_\lambda = B + \lambda B_5 \)
Chiral kinetic theory

- Kinetic equation:

\[
\frac{\partial f_\lambda}{\partial t} + \left[ e \tilde{E}_\lambda + \frac{e}{c} (\mathbf{v} \times \mathbf{B}_\lambda) + \frac{e^2}{c} (\tilde{E}_\lambda \cdot \mathbf{B}_\lambda) \Omega_\lambda \right] \cdot \nabla_p f_\lambda \cdot \nabla_p f_\lambda 
\]

\[
= 0
\]

where \( \tilde{E}_\lambda = E_\lambda - \frac{1}{e} \nabla_r \epsilon_p , \quad \mathbf{v} = \nabla_p \epsilon_p , \)

\[
\epsilon_p = v_F p \left[ 1 - \frac{e}{c} (\mathbf{B}_\lambda \cdot \Omega_\lambda) \right] 
\]

and \( \Omega_\lambda = \lambda \hbar \frac{\hat{p}}{2p^2} \) is the Berry curvature
Current and chiral anomaly

- The definitions of density and current are

\[
\rho_\lambda = e \int \frac{d^3p}{(2\pi\hbar)^3} \left[ 1 + \frac{e}{c}(B_\lambda \cdot \Omega_\lambda) \right] f_\lambda, \\
j_\lambda = e \int \frac{d^3p}{(2\pi\hbar)^3} \left[ \mathbf{v} + \frac{e}{c}(\mathbf{v} \cdot \Omega_\lambda)B_\lambda + e(\tilde{E}_\lambda \times \Omega_\lambda) \right] f_\lambda \\
+ e \nabla \times \int \frac{d^3p}{(2\pi\hbar)^3} f_\lambda \epsilon_p \Omega_\lambda,
\]

They satisfy the following anomalous relations:

\[
\frac{\partial \rho_5}{\partial t} + \nabla \cdot \mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2c} \left[ (\mathbf{E} \cdot \mathbf{B}) + (\mathbf{E}_5 \cdot \mathbf{B}_5) \right] \\
\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \frac{e^3}{2\pi^2\hbar^2c} \left[ (\mathbf{E} \cdot \mathbf{B}_5) + (\mathbf{E}_5 \cdot \mathbf{B}) \right]
\]
Consistent definition of current

• Additional Bardeen-Zumino term is needed,

\[ \delta j^\mu = \frac{e^3}{4\pi^2 \hbar^2 c} \varepsilon^{\mu\nu\rho\lambda} A_\nu^5 F_{\rho\lambda} \]


• In components,

\[ \delta \rho = \frac{e^3}{2\pi^2 \hbar^2 c^2} (A^5 \cdot B) \]
\[ \delta j = \frac{e^3}{2\pi^2 \hbar^2 c} A_0^5 B - \frac{e^3}{2\pi^2 \hbar^2 c} (A^5 \times E) \]

• Its role and implications:

– Electric charge is conserved locally \((\partial_\mu J^\mu = 0)\)

– Anomalous Hall effect is reproduced

– CME vanishes in equilibrium \((\mu_5 = -eb_0)\)
Collective modes

We search for plane-wave solutions with
\[ E' = E e^{-i\omega t + ik \cdot r}, \quad B' = B e^{-i\omega t + ik \cdot r} \]
and the distribution function\[ f_\lambda = f_\lambda^{(eq)} + \delta f_\lambda, \]
where\[ \delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + ik \cdot r} \]
The polarization vector & susceptibility tensor:
\[ P^m = i \frac{J'^m}{\omega} = \chi^{mn} E'^n \]
The plasmon dispersion relations follow from
\[ \det \left[ (\omega^2 - c^2 k^2) \delta^{mn} + c^2 k^m k^n + 4\pi\omega^2 \chi^{mn} \right] = 0 \]
Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

\[
\omega_l = \Omega_e, \quad \omega_{tr}^\pm = \Omega_e \sqrt{1 \pm \frac{\delta \Omega_e}{\Omega_e}}
\]

where the Langmuir frequency is

\[
\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi \hbar^2} \left( \mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right)}
\]

and \(\delta \Omega_e = \frac{2e\alpha v_F}{3\pi c \hbar^2} \left\{ 9\hbar^2 b_\perp^2 + \left[ \frac{2v_F}{\Omega_e^2} (B_0 \mu + B_{0,5} \mu_5) \right] \right\}^{1/2}

- 3\hbar b_\parallel - \frac{v_F \hbar^2}{4T} \sum_{\lambda=\pm} B_{0,\lambda} F \left( \frac{\mu_\lambda}{T} \right)^2 \right\}^{1/2}

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 95, 115202 (2017)]
Plasmon frequencies, $\vec{B} \perp \vec{b}$

$$b_\perp = 0.2\hbar\Omega_e/e$$

Plasmon frequencies, $\vec{B} \parallel \vec{b}$

$$b_\parallel = 0.2 \hbar \Omega_e / e$$

$$\frac{\omega}{\Omega_e}$$

$B_0^* \propto \alpha \mu b_\parallel \text{ at } T = 0$

Plasmons with $\vec{k} \neq 0$, $\vec{k} \parallel \vec{B}, \vec{B}_5$

- The longitudinal mode is sensitive to $\vec{B}_5$

\[ \frac{\omega}{\Omega_e} \]

\[ k_\parallel \propto \frac{e B_5}{\Omega_e^2} \]

\[ B_{0,5} = 0.5 \times \frac{\hbar \Omega_e^2}{(e v_F)} \]

\[ B_{0,5} = 0 \]

\[ v_F k_\parallel / \Omega_e \]

[ Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B 95, 115202 (2017)]
Summary

• Questions remain about the competition of magnetic catalysis and inverse magnetic catalysis

• Other properties need to be addressed on the lattice
  – Nucleon masses
  – Masses, spectra & decay constants of neutral pions

• Chiral anomalous effects can be tested in many branches of physics
  – Heavy-ion collisions (CME, CVE, CMW, CVE, etc.)
  – early Universe and compact stars (generation of large-scale helical magnetic fields)
  – Condensed matter physics (phase transitions, transport, collective modes, etc.)