Anomaly-driven chiral magnetic effects

Igor Shovkovy
Arizona State University
MAGNETIC FIELDS EVERYWHERE

- Current galactic magnetic fields $\sim 10^{-6}$ G
- Current magnetic fields in voids $\sim 10^{-15}$ G

- Problem of magnetogenesis in Early Universe

- Perhaps during the electro-weak phase transition
  $- 10^{20}$ to $10^{24}$ G ($\sim 1$ GeV to 100 GeV)
Dense baryonic matter

• Magnetized dense baryonic matter
  – $10^{10}$ to $10^{18}$ G (10 keV to 100 MeV)

• Magnetic field may affect
  – Competition of ground state phases
  – EoS of dense baryonic matter
  – the M-R relation of compact stars
  – Transport and emission properties
  – Evolution of supernovas & protoneutron stars

[t→ talks by Yasufumi Kojima & Teruaki Enoto]
Little Bangs

• Magnetized QGP at RHIC/LHC
  \( B \sim 10^{18} \) to \( 10^{19} \) G (\( \sim 100 \) MeV)

• Using Lienard-Wiechert potentials,

\[
eE(t, r) = \frac{e^2}{4\pi} \sum_n Z_n \frac{R_n - R_n v_n}{(R_n - R_n \cdot v_n)^3} (1 - v_n^2)
\]

\[
eB(t, r) = \frac{e^2}{4\pi} \sum_n Z_n \frac{v_n \times R_n}{(R_n - R_n \cdot v_n)^3} (1 - v_n^2)
\]

[Rafelski & Müller, PRL, 36, 517 (1976)],
[Kharzeev et al., arXiv:0711.0950],
[Skokov et al., arXiv:0907.1396],
[Voronyuk et al., arXiv:1103.4239],
[Bzdak &. Skokov, arXiv:1111.1949],
[Deng & Huang, arXiv:1201.5108]

[\( \rightarrow \) talks by Kazunori Itakura & Ron Belmont]
Dirac/Weyl materials

- High magnetic field lab
  - $10^5$ G ($\sim 100$ meV @ vF=c/300)

- Graphene

- 3D materials with Dirac/Weyl quasiparticles
  - $\text{Bi}_{1-x}\text{Sb}_x$ alloy (at $x \approx 4\%$)
  - $\text{Na}_3\text{Bi}$
  - $\text{Cd}_3\text{As}_2$
  - $\text{ZrTe}_5$
  - $\text{TaAs}$, $\text{NbAs}$, $\text{TaP}$, ...

[Z. K. Liu et al., arXiv:1310.0391]
[M. Neupane et al., arXiv:1309.7892]
[S. Borisenko et al., arXiv:1309.7978]
[X. Li et al., arXiv:1412.6543]

CHIRAL SEPARATION EFFECT

\[ \langle \tilde{j}_5 \rangle = -\frac{e\tilde{B}}{2\pi^2} \mu \]
Chirality & Anomaly

• Chirality/helicity of a massless (or ultrarelativistic) particle is (approximately) conserved

\[ \frac{\partial (n_R - n_L)}{\partial t} + \nabla \cdot \vec{j}_5 = -\frac{e^2}{16\pi^2} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda} \]

Right-handed

Left-handed

• Conservation of chiral charge is a property of massless Dirac theory (classically)

• The symmetry is anomalous at quantum level

\[ \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \Psi = \text{sign}(p_0)\gamma^5 \Psi \]
Chiral separation effect

- Slowly changing electric/chemical potential
  \[ \mu(z) = e\Phi(z) \quad \Rightarrow \quad eE_z = -\partial_z(e\Phi) = -\partial_z\mu \]

- From the anomaly relation,
  \[ \partial_z j_5^3 = -\frac{e^2}{2\pi^2}B_z E_z = \frac{e^2}{2\pi^2}B_z \partial_z\mu \]

- Suggesting that for massless fermions,
  \[ \langle \vec{j}_5 \rangle = -\frac{e\vec{B}}{2\pi^2} \mu \]

  [Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

- This can be easily derived in free theory
Landau spectrum at $B \neq 0$

- Dirac equation with massless fermions

$$\left[ i \gamma^0 \partial_0 - i \vec{\gamma} \cdot (\vec{\nabla} + ie\vec{A}) \right] \Psi = 0$$

- Energy spectrum

$$E^{(3+1)}_n(p_3) = \pm \sqrt{2n|eB| + p_3^2}$$

$s = \pm \frac{1}{2}$ (spin)

where

$$n = s + k + \frac{1}{2}$$

$k = 0, 1, 2, \ldots$ (orbital)
What if LLL is partially filled?

$E_n(p_3)$

Right-handed:

Left-handed:

What if LLL is partially filled?
Partially filled LLL

- Spin polarized LLL is chirally asymmetric
  - states with $p_3 < 0$ (and $s = \downarrow$) are R-handed
  - states with $p_3 > 0$ (and $s = \downarrow$) are L-handed
  i.e., a nonzero axial current is induced

\[
\langle \vec{j}_5 \rangle = -\frac{e \vec{B}}{2\pi^2} \mu
\]
CSE in Dirac semimetals

- Model of Dirac semimetal with a slab geometry

\[
H = \int d^3r \Psi^+ \left[ \nu_F \vec{\alpha} \cdot (-i\vec{\nabla} + e\vec{A}) + \gamma^0 m(z) \right] \Psi
\]

where \( \vec{A} = (0, Bx, 0) \) and

\[
m(z) = M \theta(z^2 - a^2) + m \theta(a^2 - z^2),
\]

with vacuum band gap: \( M \to \infty \) (broken chiral symmetry)

- Boundary conditions:

\[
\Psi_{\text{bulk}}(\vec{r}_\perp, a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, a) \quad \text{and} \quad \Psi_{\text{bulk}}(\vec{r}_\perp, -a) = \Psi_{\text{vacuum}}(\vec{r}_\perp, -a)
\]

Quantization of axial current

- Axial current density is non-uniform when $m \neq 0$

- Note that axial charge density vanishes: $\langle j_5^0 \rangle = 0$

Axial current as a standing wave?

- Recall that LLL is spin polarized

A perfect chirality flip at the boundary

CHIRAL MAGNETIC EFFECT

\[ \langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2 \pi^2} \mu_5 \]
Partially filled LLL @ $\mu_5 \neq 0$

- **Spin polarized** LLL is chirally asymmetric
  - states with $p_3 < 0$ (and $s=\downarrow$) are R-handed **electrons**
  - states with $p_3 > 0$ (and $s=\downarrow$) are L-handed **positrons**

i.e., a nonzero **electric** current is induced

\[
\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5
\]
CME in heavy ion collisions?

- Chiral charge can be produced by topological configurations in QCD

\[
\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3 x \, F_a^{\mu\nu} \tilde{F}_a^{\mu\nu}
\]

- A random fluctuation with nonzero chirality could result in

\[N_R - N_L \neq 0 \quad \Rightarrow \quad \mu_5 \neq 0\]

- This should lead to an electric current

\[\langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5\]
Dipole CME

- Dipole pattern of electric currents (or charge correlations) in heavy ion collisions

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]

[→ talk Ron Belmont]
Experimental evidence

[B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 81, 054908 (2010)]
CHIRAL MAGNETIC WAVE

\[ \langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2} \mu \]

\[ \langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2} \mu_5 \]
CMW/Quadrupole CME

- Start from a small baryon density and $B \neq 0$

$$\langle \vec{j}_5 \rangle = \frac{e\vec{B}}{2\pi^2} \mu$$

$$\langle \vec{j} \rangle = \frac{e\vec{B}}{2\pi^2} \mu_5$$

- Produce back-to-back electric currents

[Orbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
Elliptic flows of $\pi^+$ and $\pi^-$ depend on charge asymmetry:

$$\frac{dN^\pm}{d\phi} \approx \overline{N}^\pm \left[ 1 + 2v_2 \cos(2\phi) \mp A^\pm r \cos(2\phi) \right]$$

[Adamczyk et al. (STAR), Phys. Rev. Lett. 114, 252302 (2015)]
FURTHER DEVELOPMENTS

• Dynamical chiral shift
Chiral shift at $B \neq 0$

- The axial current (CSE) in free theory

$$\langle \bar{\psi} \gamma^3 \gamma^5 \psi \rangle = \frac{eB}{2\pi^2} \mu \quad \text{with} \quad \vec{B} = (0, 0, B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

+ interactions should induce a chiral shift parameter $\Delta$ associated with the condensate,

$$\delta L = \Delta \bar{\psi} \gamma^3 \gamma^5 \psi$$

This is a perturbative effect: there is no symmetry to protect $\Delta = 0$

[Gorbar, Miransky, Shovkovy, Phys. Rev. C 80, 032801(R) (2009)]
Chiral shift @ Fermi surface

- Chirality is \( \approx \) well-defined at Fermi surface \(|p_3| \gg m\)
- L-handed Fermi surface:

\[
\begin{align*}
  n > 0: \quad p_3 &= +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2} \\
  p_3 &= -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}
\end{align*}
\]

- R-handed Fermi surface:

\[
\begin{align*}
  n > 0: \quad p_3 &= -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2} \\
  p_3 &= +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}
\end{align*}
\]

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
QED in weak field (B→0)

- The result has the form

\[ \Sigma^{(1)}(p) = \gamma^3 \gamma^5 \Delta(p) + \gamma^0 \gamma^5 \mu_5(p) \]

Near Fermi surface \((p_0 \rightarrow 0, |p| \rightarrow p_F)\)

\[ \Delta(p) \approx \frac{\alpha eB \mu}{\pi m^2} \left( \ln \frac{m^2}{2 \mu(|p| - p_F)} - 1 \right) \]

\[ \mu_5(p) \approx -\frac{\alpha eB \mu}{\pi m^2} \frac{p_3}{p_F} \left( \ln \frac{m^2}{2 \mu(|p| - p_F)} - 1 \right) \]

Dispersion relations in QED

• Let us use the condition (for a small $B$)

$$\text{Det}\left[ i \bar{S}^{-1}(p) + \Sigma^{(1)}(p) \right] = 0$$

QED in strong field: $\delta p_3$

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu} \left( 0.76 + 0.49 \frac{|eB|n}{\mu^2} \right)$$

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[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D 90, 085011 (2014)]

March 8, 2016  1st CORE-U International Conference: Intense Fields and Extreme Universe, Hiroshima, Japan
How large is the asymmetry?

In QED:

$$\frac{\alpha |eB|}{\mu} \approx 0.4 \left( \frac{B}{10^{18} \text{G}} \right) \left( \frac{100 \text{MeV}}{\mu} \right) \text{MeV/c}$$

In QCD:

$$\frac{\alpha_s |eB|}{\mu} \approx 10 \left( \frac{B}{10^{18} \text{G}} \right) \left( \frac{400 \text{MeV}}{\mu} \right) \text{MeV/c}$$

may have some observable consequences...
FURTHER DEVELOPMENTS

• Anomalous Maxwell equations for chiral plasmas

[Boyarsky, Gorbar, Ruchayskiy, Rudenok, Shovkovy, Vilchinskii, to appear]
Magnetic field/helicity

- Magnetic helicity evolution (inverse cascade) in the Early Universe

\[ \nabla \times \vec{B} = \frac{4\pi}{c} \left( \sigma \vec{E} + C_5 \mu_5 \vec{B} \right) + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \]

\[ \nabla \cdot \vec{E} = 4\pi \rho, \quad \nabla \cdot \vec{B} = 0 \]

- For specific helicity eigenmodes:

\[ \frac{d B_k}{dt} \propto \frac{c k}{\sigma} \left( 4\pi C_5 \mu_5 - c k \right) B_k \]

[Vilenkin, Phys. Rev. D22, 3080 (1980)]
[Joyce & Shaposhnikov, astro-ph/9703005]
[Giovannini & Shaposhnikov, hep-ph/9710234]
[Boyarsky et al., arXiv:1109.3350]
[Tashiro et al., arXiv:1206.5549]
[Manuel et al., arXiv:1501.07608]
[Buividovich et al., arXiv:1509.02076]
[Hirono et al., arXiv:1509.07790]
Feedback on $\mu_5(t)$

- Constraint of the total helicity conservation
  
  $$h_{\text{fermion}} = h_0 - h_{\text{gauge}}$$

- Common “homogeneous” approximation:
  
  $$n_5(\vec{x}, t) \approx \langle n_5(\vec{x}, t) \rangle_{\text{space}}, \quad \mu_5(t) \approx \frac{3c^3}{T^2} \langle n_5(\vec{x}, t) \rangle_{\text{space}}$$

- In other words, the value of $\mu_5$ remains constant on distance scales
  
  $$\Delta x \sim (k_{\text{crit}})^{-1} \sim (\mu_5)^{-1}$$
Magnetic field/helicity

- Magnetic helicity is transferred from short to long-wavelengths modes, while the value of $\mu_5$ decreases.

From [Hirono et al., arXiv:1509.07790]

From [Boyarsky et al., arXiv:1109.3350]
Open questions

• Will the cascade survive if there are variations of order $\delta \mu_5$ on distance scales $(k_{\text{crit}})^{-1}$?

• How large $\delta \mu_5$ can be tolerated?

• Will dynamical fluctuations of $\mu_5$ stay under control?

• How to account for inhomogeneities in the anomalous Maxwell equations?

$$n_\lambda(\vec{x}, t) = \ ? \quad \vec{j}_\lambda(\vec{x}, t) = \ ?$$

• How to obtain equations for $\mu(t, x)$ and $\mu_5(t, x)$?
Framework

- Chiral kinetic theory as a starting point:

\[
\frac{\partial f_\lambda}{\partial t} + \frac{1}{1 + \vec{\Omega}_\lambda \cdot \vec{B}} \left[ \left( \vec{E} + \vec{v} \times \vec{B} + (\vec{E} \cdot \vec{B})\vec{\Omega}_\lambda \right) \cdot \frac{\partial f_\lambda}{\partial \vec{p}} + (\vec{v} + \vec{E} \times \vec{\Omega}_\lambda + (\vec{v} \cdot \vec{\Omega}_\lambda)\vec{B}) \cdot \frac{\partial f_\lambda}{\partial \vec{x}} \right] = I_{\text{coll}}
\]

where

\[
f_\lambda = f^{(0)}_\lambda + f^{(1)}_\lambda + f^{(2)}_\lambda + \ldots
\]

\[
f^{(0)}_\lambda = \frac{1}{\exp\left(\frac{cp - \mu_\lambda}{T}\right) + 1}
\]

is an expansion in powers of e-m field & \( \vec{\nabla} \mu_\lambda, \partial_t \mu_\lambda \)

[Boyarsky, Gorbar, Ruchayskiy, Rudenok, Shovkovy, Vilchinskii, to appear]
Equations for chemical potentials

- Resulting equation of motion for $\mu_\lambda$:

$$\frac{\partial n_\lambda^{(0)}}{\partial \mu_\lambda} \left( \frac{\partial \mu_\lambda}{\partial t} + \frac{e \tau c^2}{3} \nabla \cdot \vec{E}_\lambda \right) + \frac{e \tau \mu_\lambda}{3 \pi^2 c} \left( \frac{\vec{E}_\lambda \cdot \partial \mu_\lambda}{\partial \vec{x}} \right) = \frac{\lambda e^2}{4 \pi^2 c} (\vec{E}_\lambda \cdot \vec{B})$$

where $n_\lambda^{(0)} = \frac{\mu_\lambda^3 + \pi^2 T^2 \mu_\lambda}{3 \pi^2 c^3}$ and $\vec{E}_\lambda = \vec{E} - \frac{1}{e} \frac{\partial \mu_\lambda}{\partial \vec{x}}$

The corresponding equations for the currents:

CME drift & diffusion

$$\vec{j} = \frac{e \mu_5 \vec{B}}{2 \pi^2 c} + \frac{e \tau T^2}{9 c} \left( 1 + \frac{3 (\mu^2 + \mu_5^2)}{\pi^2 T^2} \right) \left( e \vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right) + \frac{e \tau^2 \mu}{3 \pi^2} \left( e \vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right) \times \vec{B} + \vec{j}_{\text{new}}$$

$$\vec{j}_5 = \frac{e \mu \vec{B}}{2 \pi^2 c} - \frac{e \tau T^2}{9 c} \left( 1 + \frac{3 (\mu^2 + \mu_5^2)}{\pi^2 T^2} \right) \frac{\partial \mu_5}{\partial \vec{x}} + \frac{2 e \tau \mu_5 \mu}{3 \pi^2 c} \left( e \vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right) + \vec{j}_{5,\text{new}}$$

CSE diffusion

CESE
New types of currents

- **New contribution to the electric current:**

\[
\vec{j}_{\text{new}} = -\frac{2\pi \mu \mu_5}{3 \pi^2 c} \frac{\partial \mu_5}{\partial \vec{x}} - \frac{e \pi^2 \mu_5}{3 \pi^2} \left( \frac{\partial \mu_5}{\partial \vec{x}} \times \vec{B} \right) - \frac{e \pi^2 \left( 3\mu^2 + 3\mu_5^2 + \pi^2 T^2 \right)}{9 \pi^2 c} \frac{\partial \vec{E}}{\partial t}
\]

| Chiral diffusion | Hall diffusion |

- **New contribution to the chiral current:**

\[
\vec{j}_{5,\text{new}} = -\frac{e \pi^2 \mu}{3 \pi^2} \left( \frac{\partial \mu_5}{\partial \vec{x}} \times \vec{B} \right) + \frac{e \pi^2 \mu_5}{3 \pi^2} \left( e \vec{E} - \frac{\partial \mu}{\partial \vec{x}} \right) \times \vec{B} - \frac{2e \pi^2 \mu \mu_5}{3 \pi^2 c} \frac{\partial \vec{E}}{\partial t}
\]

| Chiral Hall diffusion | Chiral Hall effect |

- **There is also a term**

\[
\propto \frac{e \pi}{6 \pi^2} \left( \vec{E} \times \frac{\partial \mu}{\partial \vec{x}} \right)
\]
Question about Hall current

• Note that we had

$$\vec{j}^{\text{Hall}} = \frac{e^2 \tau^2 \mu}{3 \pi^2} \vec{E} \times \vec{B}$$

• Why is this proportional to $\tau^2$?

• What do you observe in the usual experimental setup ($j_y=0$ and $j_x \neq 0$)?
Enforcing $j_y = 0$ gives

$$a \tau E_y = b \tau^2 E_x B_z$$

Then, in the approximation used,

$$j_x = a \tau E_x + b \tau^2 E_y B_z = \left( \frac{a \tau}{b \tau^2 B_z} \right) E_y + b \tau^2 E_y B_z \approx \frac{a^2}{b B_z} E_y$$
Plasma drift

Plasma as a whole experiences a non-dissipative drift. Can this be included in $f_\lambda^{(0)}$?

Why should plasma drift?

Consider $\vec{E} \perp \vec{B}$ (with $E < B$):

$$q \vec{E} \quad q \vec{v} \times \vec{B}$$

$$q \vec{v} \times \vec{B} \quad q \vec{E}$$
Frames of reference

Another viewpoint

Drifting frame:

Lab frame:
\( f^{(0)}_\lambda \) for magnetized plasma

- Consider a special case
  - Plasma consists of only e-m charged degrees of freedom
  - Fields so that \( \vec{E} \perp \vec{B} \) (with \( E < B \))

- No electric field in the drift frame

- Lab frame: use boosted Fermi-Dirac distribution

\[
\begin{align*}
  f^{(\text{lab})}_\lambda &= \frac{1}{\exp\left(\frac{c p - \vec{p} \cdot \vec{V}_\text{drift} - \mu_\lambda}{T}\right) + 1} \\
  \vec{V}_\text{drift} &= c \frac{\vec{E} \times \vec{B}}{B^2}
\end{align*}
\]
1st Perturbation in drifting plasma

- **Charge density**

\[ n^{(\text{lab})}_\lambda = n^{(0)}_\lambda - \tau \frac{\partial n^{(0)}_\lambda}{\partial \mu_\lambda} \left( \frac{\partial \mu_\lambda}{\partial t} + \vec{v}_{\text{drift}} \cdot \nabla \mu_\lambda \right) + \tau n^{(0)}_\lambda \left( \nabla \cdot \vec{v}_{\text{drift}} \right) + \ldots \]

- **Current density**

\[ j^{(\text{lab})}_\lambda = c n^{(0)}_\lambda \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\lambda e \mu_\lambda}{4\pi^2 c} \vec{B} + \frac{\lambda e \mu_\lambda}{4\pi^2 c} \frac{\vec{E} \cdot \vec{B}}{E_\perp} \left( \frac{B}{2E_\perp} \ln \frac{B + E_\perp}{B - E_\perp} - 1 \right) \]

\[ -\tau \frac{\partial n^{(0)}_\lambda}{\partial \mu_\lambda} \left( g_1 \frac{e(\vec{E} \cdot \vec{B})}{B^2} \vec{B} - \nabla \mu_\lambda \right) + g_2 \vec{v}_{\text{drift}} \left( \vec{v}_{\text{drift}} \cdot \nabla \mu_\lambda \right) + g_3 \frac{\partial \mu_\lambda}{\partial t} \vec{v}_{\text{drift}} \right) + \ldots \]
Drift in QGP plasma?

- In QGP, gluons play a profound role
  - Gluons are neutral and, thus, are not drifting
  - The zeroth approximation is the usual Fermi-Dirac distribution
    
    \[ f^{(0)}_{\lambda} = \frac{1}{\exp\left( \frac{cp - \mu_\lambda}{T} \right) + 1} \]

- Expansion in small e-m fields and gradients is well define
  
  \[ f_{\lambda} = f^{(0)}_{\lambda} + f^{(1)}_{\lambda} + f^{(2)}_{\lambda} + \ldots \]

- Expansion of the 1\textsuperscript{st} type (no drift) may be better
Summary

• Chiral plasmas have widespread applications
  – Heavy-ion collisions
  – Cosmology
  – Dirac/Weyl semimetals
  – Neutron stars
• Anomaly plays a profound role in such plasmas
• Many interesting chiral/anomalous effects are triggered by a magnetic field
• Experimental search for signatures is underway