Magnetism and chirality in QCD

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Role of chirality in QCD

Approximate chiral symmetry (& its breaking)

→ natural order parameter
→ dynamical origin of nucleon masses
→ spectrum of light mesons
→ structure of the chiral perturbation theory
→ anomalous properties
→ …
Role of magnetism in QCD

- Magnetized quark matter may exist inside compact stars
  - $10^{10}$ to $10^{16}$ G (10 keV to 10 MeV)

- Magnetized quark-gluon plasma is produced in heavy ion collisions ($\Delta t \approx 10^{-24}$ s)
  - $10^{18}$ to $10^{19}$ G ($\sim 100$ MeV)

- Magnetic field is a useful probe of nonperturbative QCD properties
  - $10^{21}$ G ($\sim 1$ GeV)
MAGNETIZED QCD VACUUM

Landau spectrum at $B \neq 0$

- Dirac equation with massless fermions

$$\left[ i \gamma^0 \partial_0 - i \vec{\gamma} \cdot (\vec{\nabla} + i e \vec{A}) \right] \Psi = 0$$

- Energy spectrum

$$E^{(3+1)}_n(p_3) = \pm \sqrt{2n|eB| + p_3^2}$$

$s = \pm \frac{1}{2}$ (spin)

where $n = s + k + \frac{1}{2}$

$k = 0, 1, 2, \ldots$ (orbital)
Magnetic catalysis: idea

• Low-energy fermion dynamics is dimensionally reduced

\[ D \rightarrow D - 2 \]

• Nonzero density of states at \( E = 0 \)

\[
\frac{dn}{dE} \bigg|_{E \rightarrow 0} = \frac{|eB| N_f}{4\pi^2}
\]

• Attractive interaction \( \rightarrow \) symmetry breaking
  (reminiscent of superconductivity…)

Universality of magnetic catalysis

• Input
  – Spin-$\frac{1}{2}$ charged particles and $B\neq 0$
  – Attractive particle-antiparticle interaction

• Output
  – Dimensional reduction $D\rightarrow D-2$ @ low energies
  – particle-antiparticle bound states form
  – Symmetry breaks down
  – Dynamical mass is generated
Magnetic catalysis in QCD

- Approximate dynamical mass ($\sqrt{|eB|} \gg \Lambda_{QCD}$):

$$m_f^2 \propto |e_f B| \alpha_s^{3/2} \exp \left( -\frac{4\pi N_c}{\alpha_s (N_c^2 - 1) \ln \left( \frac{C}{\alpha_s} \right)} \right)$$

![Graph showing the relationship between $m_f^2$ and $\ln(|eB|/\Lambda_{QCD}^2)$ for different values of $N_u$ and $N_d$.]

Anisotropic confinement in QCD

• Low-energy gluodynamics \( (p \ll m_{\text{dyn}}) \):

\[
L \approx \frac{1}{2} \sum_{a=1}^{N_c^2-1} \left( \vec{E}_\perp^a \cdot \vec{E}_\perp^a + \varepsilon E_z^a E_z^a - \vec{B}_\perp^a \cdot \vec{B}_\perp^a - B_z^a B_z^a \right)
\]

where the chromo-dielectric constant is given by

\[
\varepsilon \approx 1 + \frac{\alpha_s}{6\pi} \sum_{f=1}^{N_f} \frac{|q_f B|}{m_f^2} \gg 1
\]

The new confinement scale is

\[
\lambda_{\text{QCD}} \approx m_d \left( \frac{\Lambda_{\text{QCD}}}{\sqrt{|eB|}} \right)^{\frac{11N_c-2N_f}{11N_c}}
\]

Note: \( \lambda_{\text{QCD}} \ll \Lambda_{\text{QCD}} \)  

Magnetic catalysis in QCD (lattice)

Δ(Σ_0 + Σ_ρ) / 2

\[ T = 0 \]

(Inverse) Catalysis at T≠0?

[Graph showing data points and lines for different temperatures: T=0, T=130 MeV, T=148 MeV, T=153 MeV, T=163 MeV, T=176 MeV.]

[Equation or data: Δ(Σ_u + Σ_d) / 2 vs. eB (GeV²)]

Inverse catalysis: $T_c$ vs. $B$

• Gluon screening (?), Polyakov loops (?), …

• See also [Ilgenfritz et al. Phys. Rev. D 89, 054512 (2014)]
Super-strong $B$: prediction

Complete diagram: prediction

\[ T_c \sim m_{\text{dyn}} \text{ (chiral symmetry restoration)} \]

\[ T_c^* \sim \lambda_{\text{QCD}} \text{ (deconfinement)} \]

MAGNETIZED QCD MATTER

B≠0: Chiral separation effect

• Axial current density induced by the chemical potential

\[
\langle \vec{j}_5 \rangle_{\text{free}} = \frac{e\vec{B}}{2\pi^2} \mu \quad \text{(free theory!)}
\]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

• This result is connected to the chiral anomaly relation (\(\mu \rightarrow e\Phi\))

\[
\partial_z \langle j_5^z \rangle_{\text{free}} = \frac{e^2}{2\pi^2} B_z \partial_z (e\Phi) = -\frac{e^2}{2\pi^2} B_z E_z
\]

• However, axial current gets radiative corrections
Asymmetry: LLL $\mapsto$ hLLs

- LLL is spin polarized and chirally asymmetric
  - states with $p_3<0$ (and $s=\downarrow$) are R-handed
  - states with $p_3>0$ (and $s=\downarrow$) are L-handed

- This indeed implies axial current density
Spin vs. orbital motion

• Helicity/chirality of massless (ultrarelativistic) fermions is ($\approx$) conserved

- R-handed
- L-handed

• Chirality does not change in elementary QED interactions
Asymmetry: LLL ⇒ hLLs

• What is the effect of interactions?

• To preserve chirality, particle momenta have to “flip” whenever the spin “flips”

• B-field ⇒ preferred spin orientation \( s = \downarrow \)

• Interactions ⇒ chiral asymmetry in hLLs

  - L-handed prefer \( s = \downarrow \) and, thus, \( p_3 < 0 \)
  - R-handed prefer \( s = \downarrow \) and, thus, \( p_3 > 0 \)
Chiral asymmetry

- Anticipated outcome: L- & R-handed Fermi surfaces shift in $p_3$ direction

Note: $p_\perp$ is not well-defined

$p_\perp^2$ is replaced by $2n|eB|$
Chiral shift at low energies

• Ground state expectation value of the axial current (CSE)

\[
\langle \bar{\psi} \gamma^3 \gamma^5 \psi \rangle = \frac{eB}{2\pi^2} \mu \quad \text{with} \quad \vec{B} = (0,0,B)
\]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

should induce a dynamical (chiral shift) parameter \( \Delta \) associated with the condensate,

\[
\delta L = \Delta \bar{\psi} \gamma^3 \gamma^5 \psi
\]

[Gorbar, Miransky, I.S., Phys. Rev. C 80, 032801(R) (2009)]

(\( \Delta=0 \) is not protected by any symmetry)
NJL model: quick check

- NJL model (local interaction)

\[ \Delta = -\frac{1}{2} G_{\text{int}} \left\langle \bar{\psi} \gamma^3 \gamma^5 \psi \right\rangle \approx -\frac{G_{\text{int}} eB}{4 \pi^2} \mu \]
Chiral shift @ Fermi surface

- Chirality is $\approx$ well-defined at Fermi surface ($|p_3| \gg m$)

- L-handed Fermi surface:

$$n > 0: \quad p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta \right)^2 - m^2}$$

$$p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta \right)^2 - m^2}$$

- R-handed Fermi surface:

$$n > 0: \quad p_3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta \right)^2 - m^2}$$

$$p_3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta \right)^2 - m^2}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
QED in weak field ($B \to 0$)

- The result has the form

$$\Sigma^{(1)}(p) = \gamma^3 \gamma^5 \Delta(p) + \gamma^0 \gamma^5 \mu_5(p)$$

Near Fermi surface ($p_0 \to 0$, $|p| \to p_F$)

$$\Delta(p) \approx \frac{\alpha e B \mu}{\pi m^2} \left( \ln \frac{m^2}{2 \mu(|p| - p_F)} - 1 \right)$$

$$\mu_5(p) \approx -\frac{\alpha e B \mu}{\pi m^2} \frac{p_3}{p_F} \left( \ln \frac{m^2}{2 \mu(|p| - p_F)} - 1 \right)$$

Dispersion relations in QED

Let us use the condition (for a small $B$)
\[
\det\left[ i \bar{S}^{-1}(p) + \bar{\Sigma}^{(1)}(p) \right] = 0
\]

QED in strong field

Self-energy in the Landau-level representation:

\[ \Sigma(p) = 2e^{-p_⊥ l^2} \sum_{n=0}^{\infty} (-1)^n \left( -\gamma^0 \delta \mu_n - \gamma^3 \gamma^5 \Delta_n - \gamma^0 \gamma^5 \mu_{5,n} + m_n + ... \right) [P_L n - P_L n-1] - ... \]

where \( \delta \mu_n, \Delta_n, \mu_{5,n}, \ldots \) are “projections” of the self-energy on the \( n \)th Landau level,

\[ \Delta_n(p_0, p_3) = \frac{(-1)^n l^2}{8\pi} \int d^2 p_\perp e^{-p_\perp l^2} \left[ L_n + L_{n+1} \right] \text{Tr} \left[ \gamma^0 \bar{\Sigma}(p) \right] \]

where

\[ \bar{\Sigma}(p) = -4i \pi \alpha \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \bar{S}(k) \gamma^\nu D_{\mu\nu}(k - p) \]

QED in strong field: $\Delta_n$

Model fit:

$$\Delta_n = -\frac{\alpha |eB|}{\mu} \left( 0.53 + 0.32 \frac{|eB|}{\mu^2} n \right)$$

[N=2x10^8, N=2x10^9]

[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D 90, 085011 (2014)]
QED in strong field: $\mu_{5,n}$

Model fit:

$$\mu_{5,n} = -0.225 \frac{\alpha |eB|}{\mu} \sqrt{1 - \left(\frac{2n|eB|}{\mu^2}\right)^2}$$

[Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D 90, 085011 (2014)]
QED in strong field: $\delta p_3$

Model fit:

$$p_3 - p_3^{(0)} = \pm \frac{\alpha |eB|}{\mu} \left( 0.76 + 0.49 \frac{|eB| n}{\mu^2} \right)$$

[ Xia, Gorbar, Miransky, Shovkovy, Phys. Rev. D 90, 085011 (2014) ]
How large is the asymmetry?

In QED:

\[
\frac{\alpha |eB|}{\mu} \approx 0.4 \left( \frac{B}{10^{18} \text{G}} \right) \left( \frac{100 \text{MeV}}{\mu} \right) \text{MeV/c}
\]

In QCD:

\[
\frac{\alpha_s |eB|}{\mu} \approx 10 \left( \frac{B}{10^{18} \text{G}} \right) \left( \frac{400 \text{MeV}}{\mu} \right) \text{MeV/c}
\]
Neutrino asymmetry

• Neutrinos equilibrate with the “flow” of L-handed fermions via

\[
\nu_e \rightarrow v_e \\
\begin{array}{c}
e^{-} \\
L
\end{array} \\
\begin{array}{c}
Z \\
L
\end{array} \\
\begin{array}{c}
\nu_e \\
L
\end{array}
\]

• An asymmetric L-handed Fermi surface with

\[
\delta p_3 \sim \alpha |eB|/\mu
\]

should scatter \(\nu_e\)’s more preferably in the direction of the field
Neutrinos from protoneutron star

\[ \langle E_v \rangle \approx 5 \text{ to } 10 \text{ MeV} \]

\[ L_E \approx 2 \times 10^{51} \text{ erg/s} \]
\[ \approx 10^{57} \text{ MeV/s} \]

\[ N_{\text{tot}} \approx \frac{10^{57} \text{ MeV/s}}{5 \text{ MeV}} \cdot 20 \text{ s} \]
\[ \approx 4 \times 10^{57} \]

Pulsar kicks

- Sizeable kick carry momenta of order

\[(1000 \text{ km/s}) \times 1.4M_\text{Sun} \approx 3 \times 10^{36} \text{ kg} \cdot \text{m/s} \]

\[\approx 9 \times 10^{44} \text{ J/c} \]

\[\approx 6 \times 10^{57} \text{ MeV/c} \]

i.e., average momentum asymmetry per neutrino should be about

\[\frac{6 \times 10^{57} \text{ MeV/c}}{4 \times 10^{57}} \approx 1.5 \text{ MeV/c} \]
Total momentum estimate

• Total momentum carrier by L-handed fermions
  – QED (B=10^{18} G and \(\mu=100\) MeV):
    \[
P \sim N \delta p \sim 10^{57} \frac{\alpha |eB|}{\mu} \sim (70\ \text{km/s}) \times 1.4M_{\text{Sun}}
    \]
  – QCD (B=10^{18} G and \(\mu=400\) MeV):
    \[
P \sim N \delta p \sim 10^{57} \frac{\alpha_s |eB|}{\mu} \sim (1700\ \text{km/s}) \times 1.4M_{\text{Sun}}
    \]

• Pulsar kicks? Possible, but questions remain…
Summary

- Magnetism profoundly affects chiral properties of QCD

  - $T=0$ & $\mu=0$: Magnetic catalysis (lattice)

  - $T \neq 0$ & $\mu=0$: Inverse magnetic catalysis (lattice)

  - $T=0$ & $\mu \neq 0$: Chiral shift (compact stars)

  - $T \neq 0$ & $\mu \neq 0$: CME, CSE, … (relativistic HIC)