Research at the Tip of a Pencil

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“If anybody says he can think about quantum physics without getting giddy, that only shows he has not understood the first thing about it.”

Niels Bohr
— Relativistic systems (~ speed of light)

— Extreme densities (~5n₀, n₀ ≈ 3 × 10^{17} kg/m³)

— Extreme temperatures (~10^{12} Kelvin)

— Extreme magnetic fields (~10^{18} Gauss)
• Examples of relativistic matter
  – Electrons, protons, quarks inside compact stars (very dense matter)
  – Quark gluon plasma in heavy ion collisions (a very hot fireball)
  – Hot matter in the Early Universe (an extremely hot world)
  – Massless particles (e.g., quasiparticles in graphene and Dirac semimetals)
• Non-relativistic
  – Particles move much slower than the speed of light
  – Kinetic energies are much smaller than the rest energy

  \[ E_{\text{kin}} \ll E_{\text{rest}} : \quad E = c \sqrt{p^2 + m^2 c^2} \approx mc^2 + \frac{p^2}{2m} \]

• Relativistic
  – Particle velocities approach the speed of light
  – Kinetic energies are comparable to, or larger than \( E_{\text{rest}} \)

  \[ E_{\text{kin}} \geq E_{\text{rest}} : \quad E = c \sqrt{p^2 + m^2 c^2} \approx c p \]
SUPER-DENSE MATTER

• What happens when you squeeze matter to very high density? (e.g., neutrons inside neutron stars)

Pauli exclusion principle: fermions cannot occupy the same states (they end up filling out all quantum states from $p_{\text{min}} \approx 0$ to $p_{\text{max}} \propto \hbar n^{1/3}$)

$$p_{\text{max}} \propto 200 \left( \frac{n}{1 \text{ fm}^3} \right)^{1/3} \text{ MeV/c}$$
• What happens when you heat matter to very high temperature? (e.g., matter in heavy ion collisions)

Heat is equivalent to kinetic energy: average kinetic energy of particles is proportional to temperature:

\[ p \propto k_B T / c \sim 200 \left( \frac{k_B T}{200 \text{ MeV}} \right) \text{MeV/c} \]
Massless Particles

• Can matter be made of massless particles?

  Yes! Electron quasiparticles masquerade as massless particles in some materials (no rest mass energy)

• Examples:

  – Graphene
  – Bi$_{1-x}$Sb$_x$ alloy with $x \approx 0.03$
  – cadmium arsenide Cd$_3$As$_2$
  – potassium bismuthide Na$_3$Bi

  2D (planar) materials
  3D materials

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MAGNETIC FIELDS IN NATURE

• Strong magnetic fields are common inside compact stars
  – $10^{10}$ to $10^{15}$ Gauss

• In heavy ion collisions, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  – $10^{18}$ to $10^{19}$ Gauss

• Early Universe
  – up to $10^{24}$ Gauss

• High Magnetic Field Laboratory
  – $4.5 \times 10^5$ Gauss
SOME RESEARCH DIRECTIONS

– Dynamical origin of mass

– Interplay of symmetry and dynamics

– Exotic phases of matter at extreme densities/temperatures

– Effect of superstrong magnetic fields on vacuum/matter
“It is only slightly overstating the case to say that physics is the study of symmetry”

Philip W. Anderson

SYMMETRY & ORIGIN OF MASS
What is symmetry?

• Wikipedia:
  – sense of harmonious and beautiful proportion and balance
  – invariance under certain transformation

• Example:

original  rotate 18°  rotate 36°  rotate 54°  rotate 72°

Invariance under 72° rotation
• Underlying laws are symmetric, but the system/ground state changes under a symmetry transformation

• Symmetry may refer to “internal” symmetries (e.g. rotations in color/flavor spaces, rescaling, etc.)
Massless $(m=0)$ fermions enjoy chiral symmetry ("rotation" of left-handed and right-handed particles in flavor space)

- Introduction of a non-zero mass for fermions breaks chiral symmetry

\[ |m\rangle \propto C_1 |L\rangle + C_2 |R\rangle \approx \langle L | R \rangle \]
Symmetry of Hardons

- Hadron properties strongly suggest an approximate chiral symmetry that is broken “spontaneously”

  - Mass hierarchy of pions and nucleons ($m_\pi << m_N$)
  - Energy dependence of pion-nucleon interaction
  - Discovery of multiplets of hadrons with similar properties
  - Relationships between various interaction strengths
  - Properties of quark-gluon plasma in heavy-ion collision
How Symmetry Breaks?

• Because of strong interactions, almost massless quarks \((m \approx 5 \text{ Mev}/c^2)\) become heavy \((m \approx 300 \text{ Mev}/c^2)\)

• The analogue of water is a certain \((\text{quark-antiquark})\) Bose condensate in the physical vacuum
Some Key Points

• Interaction must be sufficiently strong

• Bose condensate fills/defines the vacuum

• Fermions (e.g., quarks, protons, etc.) become massive

• Light (massless) pions are predicted

• Mass can be “melted” away by high temperature/density
MAGNETIC CATALYSIS

IDEA OF MAGNETIC CATALYSIS

• Magnetic field *constrains* perpendicular motion of charged particles
• Even *arbitrarily* weak attractive interaction is sufficient to form bound states
• Condensate forms & symmetry breaks down
• Fermions become massive!

This is the essence of magnetic catalysis
Dynamical Mass

- A nonzero “dynamical” mass $m_{\text{dyn}}$ is generated

$$m_{\text{dyn}}^{(2D)} \propto \sqrt{\alpha} \sqrt{|eB|}, \quad \text{and} \quad m_{\text{dyn}}^{(3D)} \propto \sqrt{|eB|} e^{-C/\alpha}$$

- This happens even at the weakest interaction (“catalysis”)

- The phenomenon is universal (most model details are irrelevant)

- Dimensional reduction is the key ingredient (it makes interaction more efficient)
APPLICATIONS
It is a single atomic layer of graphite
[Novoselov et al., Science 306, 666 (2004)]

2D crystal with hexagonal lattice of carbon atoms

Interesting basic physics

Great promise for applied physics …
GRAPHENE

• 100 times stronger than steel
• great heat conductor
• great conductor of electricity
• nearly transparent

Galapad Settler retails for US$399. Photo: Galapad
**DIRAC FERMIONS**

- Charge carriers (~ electrons)
  - spin-$\frac{1}{2}$ fermions
  - have large speed, $v \approx \frac{c}{300}$
  - behave like massless particles

- In essence, they behave like relativistic particles ("Dirac fermions") in vacuum
Magnetic Catalysis in Graphene

• Prediction:
  
  A nonzero dynamical mass should be generated in a strong magnetic field

  [Khveshchenko, PRL 87, 206401 (2001)]
  [Gorbar, Gusynin, Miransky, Shovkovy, PRB 66 (2002) 045108]

• Possible complications:
  
  – many types of “Dirac” masses in 2D
  – competition with quantum Hall ferromagnetism
  – nonzero electron/hole density
  – impurities, lattice defects, ripples, etc.

• How to test this experimentally?
• General setup

– Current starts to run:

– Steady state:

– Hall conductivity $\sigma_{xy}$:

$$j_x = \sigma_{xy} E_y$$
QHE in Graphene

\[ E_n = \text{sgn}(n) \sqrt{2e\hbar v_F^2 |n| B} \]

\[ \sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{h} \left( n + \frac{1}{2} \right) \]


Theory

Experiment
ANOMALOUS QHE

- New plateaus are observed at
  \( \nu = 0 \)
  \( \nu = \pm 1 \)
  \( \nu = \pm 3 \)
  \( \nu = \pm 4 \)


[Novoselov et al., Science 315, 1379 (2007)]
[Checkelsky et al., Phys. Rev. Lett. 100, 206801 (2008)]
[Xu Du et al., Nature 462, 192 (2009)]
Is “3D Graphene” Possible?

• “Old” 3D materials with Dirac quasiparticles:
  – Bi$_{1-x}$Sb$_x$ alloy

• “New” 3D Dirac materials:
  – Na$_3$Bi [Liu et al., Science 343, 864 (2014)]
  – Cd$_3$As$_2$ [Neupane et al., Nat. Commun. 5, 3786 (2014)]
Cadmium Arsenide

3D Dirac semimetal Cd$_3$As$_2$

Dispersion

\[ k_z = \pm k_{dp} \]

E
\[ k_x \]
\[ k_y \]
\[ k_z \]
\[ k_x = k_y = 0 \]

n-doped

p-doped

Fermi surface

In the vicinity of 3D Dirac points:

\[ E = v_x k_x + v_y k_y + v_z k_z \]

[Liu et al., Science 343, 864 (2014)]
SUMMARY

• Relativistic matter in magnetic field is relevant for many branches of physics

• The underlying physics is very rich

• A partial list of phenomena
  – Magnetic catalysis
  – Chiral magnetic effect
  – Chiral shift
  – Chiral magnetic spiral
  – Magnetic properties of Dirac semimetals
  – Quantum Hall Effect in graphene
  – …
Quantum field theory in a magnetic field: From quantum chromodynamics to graphene and Dirac semimetals

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