Quantum Magnetic Phenomena: From QCD to Dirac semimetals

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INTRODUCTION
Relativistic Matter

• Examples of relativistic matter

  – **Electrons, protons, quarks** inside compact stars (white dwarfs, neutron, hybrid or quark stars)

  – **Quark gluon plasma** in heavy ion collisions \( (k_B T \sim 200 \text{ MeV} \sim 10^{12} \text{ K}) \)

  – **Hot matter** in the Early Universe \( (k_B T \sim 100 \text{ GeV at } EW\text{ transition}) \)

  – **Quasiparticles** in Dirac semimetals (graphene, \( \text{Na}_3\text{Bi}, \text{Cd}_3\text{As}_2 \) with zero mass Dirac fermions)
What Means “Relativistic”?

- Relativistic matter \((p \gg mc)\)
  \[
  E = c\sqrt{p^2 + m^2c^2} \approx cp
  \]
  compare with nonrelativistic case \((p \ll mc)\)
  \[
  E = c\sqrt{p^2 + m^2c^2} \approx mc^2 + \frac{p^2}{2m}
  \]
  - High density (e.g., in stars) leads to occupation of states with large momenta:
    \[
    p \sim \hbar n^{1/3} \approx 200 \left(\frac{n}{1\text{ fm}^3}\right)^{1/3} \text{ MeV/c}
    \]
  - High temperature (e.g., heavy ion collisions) means energetic particles,
    \[
    p \sim k_B T/c \approx 200 \left(\frac{k_B T}{200 \text{ MeV}}\right) \text{ MeV/c}
    \]
  - Vanishing mass (e.g., graphene) works too…
Magnetic Fields

• Strong magnetic fields exist inside *compact stars*
  
  – $10^{10}$ to $10^{15}$ Gauss

• In *heavy ion collisions*, positive ions generate short-lived ($\Delta t \approx 10^{-24}$ s) magnetic fields
  
  – $10^{18}$ to $10^{19}$ Gauss

• *Early Universe*
  
  – up to $10^{24}$ Gauss

• *Graphene* (High Magnetic Field Laboratory)
  
  – up to $5 \times 10^5$ Gauss
MAGNETIC CATALYSIS

Landau levels

- Fermions in magnetic field
  \[ \mathcal{L} = \overline{\Psi} i \gamma^\mu D_\mu \Psi + \text{(interactions)} \]

- Free energy spectrum
  \[ E_n^{(3+1)}(p_3) = \pm \sqrt{2n|eB| + p_3^2} \]

Where

- \( s = \pm \frac{1}{2} \) (spin)
- \( n = s + k + \frac{1}{2} \)
- \( k = 0, 1, 2, \ldots \) (orbital)
• Low-energy is due to \( n=0 \) Landau level

\[
n = 0 : \quad E_0^{(3+1)}(p_3) = \pm p_3
\]

\[
(k = 0, s = -\frac{1}{2})
\]

• This is (1+1)D spectrum!

• Propagator also looks like in (1+1)D:

\[
S(p_{\parallel}) \approx i \, e^{-p_{\perp}^2\ell^2} \left( \hat{p}_{\parallel} + m \right) \left( 1 - i \gamma^1 \gamma^2 \right), \quad \text{where} \quad \hat{p}_{\parallel} = p_0\gamma^0 - p_3\gamma^3
\]

\[
s = -\frac{1}{2} \quad \text{spin projector}
\]
• Low-energy regime is dimensionally reduced

\[ D \Rightarrow D - 2 \]

• Density of states at \( E = 0 \)

\[ \left. \frac{dn}{dE} \right|_{E \to 0} = \frac{|eB|N_f}{4\pi^2} \]

• Attractive interaction \( \rightarrow \) gap in the spectrum

(This may remind superconductivity…)

Magnetic Catalysis (physics)

- n=0: particles & anti-particles

- Bound states are energetically favorable (an energy gain of $E_b$ per pair)

- Bound states are bosons

- Bosons can (and will) occupy same zero momentum quantum state

- Bose condensate forms

- Symmetry breaking $\rightarrow$ energy (mass) gap

Dynamical Mass

- While $m_0=0$ originally, a nonzero “dynamical” mass $m_{\text{dyn}}$ is generated

$$m_{\text{dyn}}^{(2D)} \propto \sqrt{\alpha} \sqrt{|eB|}, \quad \text{and} \quad m_{\text{dyn}}^{(3D)} \propto \sqrt{|eB|} e^{-C/\alpha}$$

- This happens even at the weakest interaction (“catalysis”)

- Dimensional reduction and finite density of states at $E=0$ play the key role

- The phenomenon is largely insensitive to model details
MAGNETIC CATALYSIS IN QCD

Credit: Centre for the Subatomic Structure of Matter, University of Adelaide
QCD in a magnetic field

- Weak coupling, (semi-)perturbative regime:
  \[ \sqrt{eB} \gg \Lambda_{\text{QCD}} \]

- Running QCD coupling

\[ M_g^2 \approx \frac{\alpha_s}{\pi} \sum_f |e_f B| \]


May 26, 2014

Low Energy Challenges for High Energy Physicists, Perimeter Institute
Dynamical quark masses

- Approximate dynamical mass:

\[ m_f^2 \propto |e_f B| \alpha_s^{3/2} \exp \left( -\frac{4\pi N_c}{\alpha_s (N_c^2 - 1) \ln(C/\alpha_s)} \right) \]
Catalysis at $T=0$ (lattice)

$\Delta(\Sigma_u + \Sigma_d) / 2$

- $a=0.29$ fm
- $a=0.215$ fm
- $a=0.15$ fm
- $a=0.125$ fm
- $a=0.1$ fm
- cont. limit

$T=0$

$eB$ (GeV$^2$)

(Inverse) Catalysis at T≠0

\[ \Delta(\Sigma_u + \Sigma_d) / 2 \]

\[ \Delta \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \]

\[ eB \ (GeV^2) \]

\[ T=0 \]
\[ T=130 \ MeV \]
\[ T=148 \ MeV \]
\[ T=153 \ MeV \]
\[ T=163 \ MeV \]
\[ T=176 \ MeV \]


[Bali et al., JHEP 02 (2012) 044]
$T_c$ vs. $B$ (inverse catalysis)

$T_c$ (MeV) vs. $eB$ (GeV$^2$)

- Red: continuum
- Black: $N_t=10$
- Green: $N_t=8$
- Blue: $N_t=6$

$\bar{u}u' + \bar{d}d'$

[Bali et al., JHEP 02, 044 (2012)]
Hints of gluon screening (?)

See also [Ilgenfritz et al. Phys. Rev. D 89, 054512 (2014)]
Dirac Fermions in graphene

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Low Energy Challenges for High Energy Physicists, Perimeter Institute
Low-energy theory

- Low energy quasiparticles are \textbf{massless} Dirac fermions \((\nu_F = c/300)\)

- Spinor:

\[
\Psi_s = \begin{pmatrix}
\psi_{KAs} \\
\psi_{K Bs} \\
\psi_{K' Bs} \\
\psi_{K' As}
\end{pmatrix}
\]

- Low-energy model with U(4) global symmetry:

\[
H_0 = \nu_F \int d^2 r \overline{\Psi}_s \left( \gamma^1 \pi_x + \gamma^2 \pi_y \right) \Psi_s
\]

Quantum Hall Effect

\[ E_n = \pm \sqrt{2\hbar v_F^2 n|eB|} \]

4-fold degenerate levels

\[ n = 0, 1, 2, 3 \]

\[ E_F \]

\[ \sigma_{xy} = \nu \frac{e^2}{h} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h} \]

Anomalous QHE

• New plateaus at
  \( \nu = 0 \)
  \( \nu = \pm 1 \)
  \( \nu = \pm 3 \)
  \( \nu = \pm 4 \)

• Some Landau level degeneracy is lifted

Zhang et al., PRL 96, 136806 (2006)

[Novoselov et al., Science 315, 1379 (2007)]
[Checkelsky et al., Phys. Rev. Lett. 100, 206801 (2008)]
[Xu Du et al., Nature 462, 192 (2009)]
Magnetic Catalysis in Graphene

- Charge carriers are massless Dirac fermions
- Spectrum in magnetic field:
  \[ E_n = \pm \sqrt{2\hbar v_F^2 n|eB|} \]
- Degenerate E=0 level with particles & holes
- Electron-hole (excitonic) pairing occurs
- \( m_{\text{dyn}} \neq 0 \) is generated
- In qualitative agreement with experiment

NONZERO DENSITY: CHIRAL SHIFT
Helicity/Chirality

• Helicities of massless (or ultra-relativistic) particles are (approximately) conserved

Righ-handed

Left-handed

• Conservation of chiral charge is a property of massless Dirac theory (classically)

• The symmetry is anomalous at quantum level
“Continuity” equation

- Continuity equation for the chiral charge

\[ \frac{\partial \rho_5}{\partial t} - \nabla \cdot \vec{j}_5 = - \frac{e}{2\pi^2} (\vec{B} \cdot \vec{E}) \]

- Among its consequences are the relations:

\[ \langle \vec{j}_5 \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu \quad \quad \quad \langle \vec{j} \rangle = \frac{e^2 \vec{B}}{2\pi^2} \mu_5 \]

- These are key relations of the chiral magnetic effect

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]
• Any radiative corrections to CSE?

\[
\langle \vec{j}_5 \rangle = - \frac{e \vec{B}}{2\pi^2} \mu + \ldots
\]

[Metlitski & Zhitnitsky, Phys Rev D 72, 045011 (2005)]

• Is there a dynamical parameter \( \Delta \) ("chiral shift") associated with this condensate?

\[
\mathcal{L} = \mathcal{L}_0 + \Delta \overline{\psi} \gamma^3 \gamma^5 \psi
\]

[\( \Delta=0 \) is not protected by any symmetry]

• Yes! Chiral shift is dynamically generated

Self-energy at $B \neq 0$

$$ \Sigma^{(1)}(p) = -4i\pi \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \bar{S}^{(1)}(k) \gamma^\nu D_{\mu\nu}(k - p) $$

- The result has the form

$$ \Sigma^{(1)}(p) = \gamma^3 \gamma^5 \Delta + \gamma^0 \gamma^5 \mu_5(p) $$

where

$$ \Delta \approx \frac{\alpha eB \mu}{\pi m^2} \left( \ln \frac{m^2}{2\mu(|p| - p_F)} - 1 \right) $$

$$ \mu_5(p) \approx -\frac{\alpha eB \mu}{\pi m^2} \frac{p_3}{p_F} \left( \ln \frac{m^2}{2\mu(|p| - p_F)} - 1 \right) $$
Chiral shift and Fermi surface

- QED: Fermi surface of L-handed (R-handed fermions is shifted in negative (positive) z-direction

\[
\text{Det}\left[i\mathcal{S}^{-1}(p) + \bar{\Sigma}^{(1)}(p)\right] = 0
\]

Corrections to axial current

- Final result (loops + counterterms)

\[
\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{2\mu}{m} + \ln \frac{m_\gamma}{m^2} + \frac{4}{3} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{2^{3/2} \mu}{m_\gamma} - \frac{11}{12} \right)
\]

- Unphysical dependence on photon mass because infrared physics with

\[
m_\gamma \leq |k_0|, \quad |k_3| \leq \sqrt{eB}
\]

not captured properly

- Note: similar problem exists in calculation of Lamb shift

DIRAC SEMIMETALS

Credits: Borisenko et al., arXiv:1309.7978
Dirac semimetals

• Solid state materials with Dirac quasiparticles:
  – $\text{Bi}_{1-x}\text{Sb}_x$ alloy

• “New” 3D Dirac materials (ARPES):
  – $\text{Na}_3\text{Bi}$ [Z. K. Liu et al., arXiv:1310.0391]
  – $\text{Cd}_3\text{As}_2$ [M. Neupane et al., arXiv:1309.7892]
    [S. Borisenko et al., arXiv:1309.7978]
Cadmium arsenide

3D Dirac semimetal Cd$_3$As$_2$

$E = E_f + \Delta E$

$E = E_f - \Delta E$

$E = E_f$

$E = E_f \pm \Delta E$

$n$-doped

$p$-doped

[S. Borisenko et al., arXiv:1309.7978]
In the vicinity of 3D Dirac points:

\[ E = v_x k_x + v_y k_y + v_z k_z \]

[Z. K. Liu et al., arXiv:1310.0391]
$\mu=0$: Semimetal $\rightarrow$ Insulator

- Doping $\rightarrow$ neutrality point ($\mu=0$)

- Magnetic field $B$ and small temperature: mass gap generation

$$m_{\text{dyn}} \sim 10^{-3} \sqrt{|eB|} \approx 8 \times 10^{-3} \sqrt{B[T]} \text{ eV} \approx 90 \sqrt{B[T]} \text{ K}$$

(assuming that coupling constant $\alpha \approx 1$)

- Experimental signatures are expected in transport measurements
**µ≠0: Dirac → Weyl metal**

- Hamiltonian of a Dirac semimetal

\[
H^{(D)} = \int d^3r \bar{\psi} \left[-i v_F (\vec{\gamma} \cdot \vec{\nabla}) - \mu_0 \gamma^0 \right] \psi + H_{\text{int}}
\]

cf. Weyl semimetal

\[
H^{(W)} = \int d^3r \bar{\psi} \left[-i v_F (\vec{\gamma} \cdot \vec{\nabla}) - (\vec{b} \cdot \vec{\gamma}) \gamma^5 - \mu_0 \gamma^0 \right] \psi + H_{\text{int}}
\]

- In a Dirac semimetal, a nonzero chiral shift \( \vec{b} \) will be induced when \( B ≠ 0 \), i.e.,

\[
\vec{b} \propto -\frac{g}{v_F^2 c} \mu_0 e \vec{B}
\]

Negative magnetoresistance

- \( \rho_{33} \) is expected to decrease with \( B \) because
  \[
  \sigma_{33} \propto B^2 \quad \text{(weak } B) \quad \text{[Son & Spivak, Phys. Rev. B 88, 104412 (2013)]}
  \]
  \[
  \sigma_{33} \propto B \quad \text{(strong } B) \quad \text{[Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]}
  \]
- Experimental confirmation? [Kim, et al., PRL 111, 246603 (2013)]
$\rho_{33} = \frac{1}{\sigma_{33}}$

Note: $\sigma_{33} = \sigma_{33}^{(LLL)} + \sigma_{33}^{(HLL)}$, where $\sigma_{33}^{(LLL)} = \frac{e^2 v_F |eB|}{4\pi^2 c \Gamma_0}$

\[\rho_{33}^{(HLL)} = \frac{1}{\sigma_{33}^{(HLL)}}\]

\[\rho_{33}^{(LLL)} = \frac{1}{\sigma_{33}^{(LLL)}}\]
$\rho_{12}$

Transverse off-diagonal $\rho_{12}$

$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\text{anom}}$$

where

$$\sigma_{12,\text{anom}} = \frac{e^2 b}{2\pi^2}$$

Summary

• Relativistic matter in magnetic fields is relevant for many branches of physics

• The underlying physics is conceptually rich

• A short list of recent developments
  – Magnetic catalysis (QCD, graphene, Dirac metals)
  – Chiral magnetic/separation effect (QCD, Dirac metals)
  – Chiral shift (QED/QCD plasma, Dirac/Weyl metals)
  – Chiral magnetic spiral …
  – and many others …