Chiral separation effect: Theoretical challenges and applications

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INTRODUCTION
• Helicities of (ultra-relativistic) massless particles are (approximately) conserved

• Conservation of chiral charge is a property of massless Dirac theory (classically)

• The symmetry is anomalous at quantum level
Chiral magnetic effect

- Chiral charge is produced by topological QCD configurations

\[
\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \, F_{\mu\nu} \tilde{F}_{\mu\nu}^a
\]

- Random fluctuations with nonzero chirality in each event

\[N_R - N_L \neq 0 \implies \mu_5 \neq 0\]

- Driving electric current

\[\langle \tilde{j} \rangle = \frac{e^2}{2\pi^2} \frac{\tilde{B}}{\mu_5}\]
Heavy ion collisions

• Dipole pattern of electric currents (charge correlations) in heavy ion collisions

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]
Experimental evidence

[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]
[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]
Chiral separation effect

- Axial current density induced by electric chemical potential

\[ \langle j^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu \] (free theory!)

- Exact result (is it?), which follows from chiral anomaly relation

- No radiative correction expected…
Possible implication

- Seed chemical potential ($\mu$) induces axial current
  \[
  \left\langle j_5^3 \right\rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu
  \]
- Leading to separation of chiral charges:
  $\mu_5 > 0$ (one side) \& $\mu_5 < 0$ (another side)
- In turn, chiral charges induce back-to-back electric currents through
  \[
  \left\langle j_3^3 \right\rangle_{\text{free}} = \frac{e^2B}{2\pi^2} \mu_5
  \]
Quadrupole CME

- Start from a small baryon density and $B \neq 0$

- Produce back-to-back electric currents

  [Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
Experimental evidence

• Elliptic flows of $\pi^+$ and $\pi^-$ depend on charge asymmetry:

$$\frac{dN^\pm}{d\phi} \approx N^\pm [1 + 2v_2 \cos(2\phi) \mp A^\pm r \cos(2\phi)]$$

Part 2

CHIRAL SHIFT

Motivation

• Any additional consequences of the CSE relation?

\[ \langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu \quad (\text{free theory!}) \]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

• Any dynamical parameter \( \Delta \) (“chiral shift”) associated with this condensate?

\[ \mathcal{L} = \mathcal{L}_0 + \Delta \overline{\psi} \gamma^3 \gamma^5 \psi \]

• Note: \( \Delta = 0 \) is not protected by any symmetry
NJL model: YES

- NJL model (local interaction)

\[ \mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle \]  

\[ m = m_0 - G_{\text{int}} \langle \bar{\psi} \psi \rangle \]  

\[ \Delta = -\frac{1}{2} G_{\text{int}} \langle j^3_5 \rangle \]

(“effective” chemical potential)

(dynamical mass)

(chiral shift parameter)
Chiral shift @ Fermi surface

• Chirality is \( \approx \) well defined at Fermi surface (\(|k^3| \gg m\))

• L-handed Fermi surface:

\[
\begin{align*}
  n = 0 : \quad k^3 &= +\sqrt{(\mu - s_\perp \Delta)^2 - m^2} \\
  n > 0 : \quad k^3 &= +\sqrt{(\sqrt{\mu^2 - 2n|eB| - s_\perp \Delta})^2 - m^2} \\
  k^3 &= -\sqrt{(\sqrt{\mu^2 - 2n|eB| + s_\perp \Delta})^2 - m^2}
\end{align*}
\]

• R-handed Fermi surface:

\[
\begin{align*}
  n = 0 : \quad k^3 &= -\sqrt{(\mu - s_\perp \Delta)^2 - m^2} \\
  n > 0 : \quad k^3 &= -\sqrt{(\sqrt{\mu^2 - 2n|eB| - s_\perp \Delta})^2 - m^2} \\
  k^3 &= +\sqrt{(\sqrt{\mu^2 - 2n|eB| + s_\perp \Delta})^2 - m^2}
\end{align*}
\]

[Gorbar, Miransky, Shovkovy, PRD 83 (2011) 085003]
\[ \Sigma^{(1)}(p) = -4i\pi \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \overline{S}^{(1)}(k) \gamma^\nu D_{\mu\nu}(k - p) \]

- The result has the form

\[ \Sigma^{(1)}(p) = \gamma^3\gamma^5\Delta + \gamma^0\gamma^5\mu_5(p) \]

where

\[ \Delta \approx \frac{\alpha eB \mu}{\pi m^2} \left( \ln \frac{m^2}{2\mu(|p| - p_F)} - 1 \right) \]

\[ \mu_5(p) \approx -\frac{\alpha eB \mu}{\pi m^2} \frac{p^3}{p_F} \left( \ln \frac{m^2}{2\mu(|p| - p_F)} - 1 \right) \]
Dispersion relations in QED

- Let us use the condition (for a small $B$)

$$\text{Det}\left[i \tilde{S}^{-1}(p) + \Sigma^{(1)}(p)\right] = 0$$

Chiral shift vs. axial anomaly

• Does the chiral shift modify the axial anomaly relation?

• Using point splitting method, one derives

$$
\langle \partial_\mu j_5^\mu (u) \rangle = -\frac{e^2 \epsilon^\beta \mu \lambda \sigma F_{\alpha \mu} F_{\lambda \sigma} \epsilon^\alpha \epsilon^\beta}{8\pi^2 \epsilon^2} \left( e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right)
$$

$$
\rightarrow -\frac{e^2}{16\pi^2} \epsilon^\beta \mu \lambda \sigma F_{\beta \mu} F_{\lambda \sigma} \quad \text{for} \quad \epsilon \to 0
$$


• Therefore, the chiral shift does **not** affect the conventional axial anomaly relation
Axial current

• However, the chiral shift does give a contribution to the axial current

• In the point splitting method, one has

\[
\langle j_5^\mu \rangle_{\text{singular}} = - \frac{\Delta}{2\pi^2 \varepsilon^2} \delta^{3}_{\mu} \approx \frac{\Lambda^2 \Delta}{2\pi^2} \delta^{3}_{\mu}
\]


• This is consistent with the NJL calculations

• Since \( \Delta \sim g\mu eB/\Lambda^2 \), the correction to the axial current is finite
Part 3

RADIATIVE CORRECTIONS TO AXIAL CURRENT

Axial current in QED

- Lagrangian density

\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \left( i \gamma^\mu D_\mu + \mu \gamma^0 - m \right) \psi + (\text{counterterms}) \]

- Axial current

\[ \langle j_5^3 \rangle = -Z_2 \text{tr} \left[ \gamma^3 \gamma^5 G(x,x) \right] \]

- To leading order in coupling \( \alpha = e^2/(4\pi) \)

\[ G(x,y) = S(x,y) + i \int d^4 u d^4 v \, S(x,u) \Sigma(u,v) S(v,y) \]
Expansion in external field

- Use expansion of $S(x,y)$ in powers of $A_{\mu}^{\text{ext}}$

- To leading order in coupling,

$$\left\langle j_5^3 \right\rangle_0 = \quad \rightarrow$$

- The radiative correction is

$$\left\langle j_5^3 \right\rangle_\alpha = \quad \rightarrow + \quad \rightarrow + \quad \rightarrow$$
Alternative form of expansion

- Expand $S(x, y) = e^{i\Phi(x, y)} S(x - y)$ in field

$$S(x, y) = S^{(0)}(x - y) + S^{(1)}(x - y) + i\Phi(x, y) S^{(0)}(x - y)$$

- The Schwinger phase (in Landau gauge)

$$\Phi(x, y) = -\frac{eB}{2} (x_1 + y_1)(x_2 - y_2)$$

- Note: the phase is not translation invariant
Translation invariant parts

- Fourier transforms

\[
\overline{S}^{(0)}(k) = i \frac{(k_0 + \mu)\gamma^0 - \mathbf{k} \cdot \gamma + m}{\left((k_0 + \mu + i \epsilon \text{sign}(k_0))^2 - \mathbf{k}^2 - m^2\right)}
\]

\[
\overline{S}^{(1)}(k) = -\gamma^1 \gamma^2 eB \frac{(k_0 + \mu)\gamma^0 - k_3\gamma^3 + m}{\left[\left((k_0 + \mu + i \epsilon \text{sign}(k_0))^2 - \mathbf{k}^2 - m^2\right)^2\right]}
\]

- Note the singularity near the Fermi surface…
Fermi surface singularity

- “Vacuum” + “matter” parts

\[
\frac{1}{\left[ \left( k_0 + \mu + i \epsilon \text{sign}(k_0) \right)^2 - k^2 - m^2 \right]^n} = "\text{Vac.}" + "\text{Mat.}" \\
\]

where

"\text{Vac.}" = \[ \frac{1}{\left[ \left( k_0 + \mu \right)^2 - k^2 - m^2 + i \epsilon \right]^n} \]

"\text{Mat.}" = \[ \frac{2 \pi i (-1)^{n-1}}{(n-1)!} \theta(|\mu| - |k_0|) \theta(-k_0 \mu) \delta^{(n-1)} \left[ \left( k_0 + \mu \right)^2 - k^2 - m^2 \right] \]
Axial current (0\textsuperscript{th} order)

- From definition

\[ \langle j_5^3 \rangle_0 = - \int \frac{d^4 k}{(2\pi)^4} \text{tr} \left[ \gamma^3 \gamma^5 S^{(1)}(k) \right] \]

- After integrating over energy

\[ \langle j_5^3 \rangle_0 = - \frac{eB \text{sign}(\mu)}{4\pi^3} \int d^3 k \delta(\mu^2 - k^2 - m^2) \]

and finally

\[ \langle j_5^3 \rangle_0 = - \frac{eB \text{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2} \]

- Note the role of the Fermi surface (!)
Conventional wisdom

• Only the lowest (n=0) Landau level contributes

\[ \langle j_5^3 \rangle_0 = \frac{eB}{4\pi^2} \int dk_3 \left[ \theta(-\mu - \sqrt{k_3^2 + m^2}) - \theta(\mu - \sqrt{k_3^2 + m^2}) \right] \]

giving same answer

\[ \langle j_5^3 \rangle_0 = -\frac{eB \text{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2} \]

• There are no contributions from higher Landau levels (n≥1)

• There is a connection with the index theorem
Two facets

- Two ways to look at the same result

$B \rightarrow 0$

$B \neq 0$
Radiative correction

- Original two-loop expression

\[
\langle j^3_S \rangle_\alpha = \frac{32\pi \alpha e B}{\Lambda^2} \int \frac{d^4p d^4k}{(2\pi)^8} \left[ \frac{1}{(P - K)^2} \left( \frac{(k_0 + \mu)[3(p_0 + \mu)^2 + p^2 + m^2] - 4(p_0 + \mu)(p \cdot k + 2m^2)}{(P^2 - m^2)^3(K^2 - m^2)} \right) - \frac{(k_0 + \mu)[3(p_0 + \mu)^2 - p^2 + 3m^2] - 2(p_0 + \mu)(p \cdot k)}{3(P^2 - m^2)^2(K^2 - m^2)^2} \right] \langle j^3_S \rangle_c \text{t.} 
\]

- After integration by parts

\[
\langle j^3_S \rangle_\alpha = \frac{64i\pi^2 \alpha e B}{\Lambda^2} \int \frac{d^4p d^4k}{(2\pi)^8} \left[ \frac{(k_0 + \mu)(p_0 + \mu) - p \cdot k - 2m^2}{(P - K)^2(K^2 - m^2)} \delta' \left[ \mu^2 - m^2 - p^2 \right] \delta(p_0) \right] + \frac{3(p_0 + \mu)^2 - 3(k_0 + \mu)(p_0 + \mu) + p^2 - p \cdot k + 3m^2}{3(P - K)^2(K^2 - m^2)^2} \delta \left( \mu^2 - m^2 - k^2 \right) \delta(k_0) + \langle j^3_S \rangle_c \text{t.} 
\]
Result \((m \ll \mu)\)

- **Loop contribution**

\[
f_1 + f_2 + f_3 = \frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{\Lambda}{2\mu} + \frac{11}{12} \right) + \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{\Lambda}{2^{3/2} \mu} + \frac{1}{6} \right)
\]

- **Counterterm**

\[
\langle j_5^3 \rangle_{ct} = -\frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{\Lambda}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{9}{4} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{\Lambda}{m_\gamma} - \frac{3}{4} \right)
\]

- **Final result**

\[
\langle j_5^3 \rangle_{\alpha} = -\frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{2\mu}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{4}{3} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{2^{3/2} \mu}{m_\gamma} - \frac{11}{12} \right)
\]
Sign of nonperturbative physics

- Unphysical dependence on photon mass

\[
\langle j_5^3 \rangle_\alpha = -\frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{2\mu}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{4}{3} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{2^{3/2} \mu}{m_\gamma} - \frac{11}{12} \right)
\]

- Infrared physics with

\[
m_\gamma \leq |k_0|, |k_3| \leq \sqrt{|eB|}
\]

not captured properly

- Note: similar problem exists in calculation of Lamb shift
Nonperturbative effects (?)

• Perpendicular momenta cannot be defined with accuracy better than
  \[ |\Delta k_\perp|_{\text{min}} \sim \sqrt{eB} \]
  (In contrast to the tacit assumption in using expansion in powers of \( B \)-field)

• Screening effects provide a natural infrared regulator
  \[ m_\gamma \Rightarrow \sqrt{\alpha \mu} \]
  (Formally, this goes beyond the leading order in coupling)
Part 4

APPLICATIONS
Protoneutron stars

• Trapped neutrinos scatter on left-handed fermions near Fermi surface:
  \[ \nu_e + e^- \rightarrow \nu_e + e^- \]

• Average momentum of scattered neutrinos
  \[ \langle \vec{p}_\nu \rangle \propto -\vec{B} \]

• Pusar kick (?)
Dirac semimetals

• Solid state materials with Dirac quasiparticles:
  – Bi$_{1-x}$Sb$_x$ alloy

@ x ~ 3-4%: Dirac metal in 3+1 d
Quantum critical point

3~4 : Inversion of the band at L

• “New” 3D Dirac materials (ARPES):
  – Na$_3$Bi  [Z. K. Liu et al., arXiv:1310.0391]
  – Cd$_3$As$_2$ [M. Neupane et al., arXiv:1309.7892]
    [S. Borisenko et al., arXiv:1309.7978]
Cadmium arsenide

3D Dirac semimetal Cd₃As₂

$n$-doped

$p$-doped

Dispersions:

- $k_z = \pm k_{DP}$
- $k_x = \pm k_{DP}$
- $k_x = k_y = 0$

Fermi surface:

[S. Borisenko et al., arXiv:1309.7978]
In the vicinity of 3D Dirac points:

\[ E = \nu_x k_x + \nu_y k_y + \nu_z k_z \]

[Z. K. Liu et al., arXiv:1310.0391]
Dirac into Weyl semimetal

- Hamiltonian of a Dirac semimetal

\[ H^{(D)} = \int d^3 r \overline{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{\nabla}) - \mu_0 \gamma^0 \right] \psi + H_{\text{int}} \]

cf. Weyl semimetal

\[ H^{(W)} = \int d^3 r \overline{\psi} \left[ -i v_F (\vec{\gamma} \cdot \vec{\nabla}) - (\vec{b} \cdot \vec{\gamma}) \gamma^5 - \mu_0 \gamma^0 \right] \psi + H_{\text{int}} \]

- In a Dirac semimetal, a nonzero chiral shift \( \vec{b} \) will be induced when \( B \neq 0 \), i.e.,

\[ \vec{b} \propto -\frac{g}{v_F^2 c} \mu_0 e \vec{B} \]

Negative magnetoresistance

• $\rho_{33}$ is expected to decrease with $B$ because

  \[ \sigma_{33} \propto B^2 \quad \text{(weak $B$)} \quad \text{[Son & Spivak, Phys. Rev. B 88, 104412 (2013)]} \]

  \[ \sigma_{33} \propto B \quad \text{(strong $B$)} \quad \text{[Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]} \]

• Experimental confirmation? \text{[Kim, et al., PRL 111, 246603 (2013)]}
Longitudinal resistivity

\[ \rho_{33}(\text{LLL}) = \frac{1}{\sigma_{33}(\text{LLL})} \]

\[ \rho_{33}(\text{HLL}) = \frac{1}{\sigma_{33}(\text{HLL})} \]

\[ \Gamma_0 = \Gamma = 0.1 \mu \]

- Note: \( \sigma_{33} = \sigma_{33}(\text{LLL}) + \sigma_{33}(\text{HLL}) \), where \( \sigma_{33}(\text{LLL}) = \frac{e^2 v_F |eB|}{4\pi^2 c \Gamma_0} \)
Transverse resistivity $\rho_{11}$

$\theta_{11}^2 e^2 \mu / v_F$

$\nu_F^2 |eB| / (c\mu^2)$

Transverse off-diagonal $\rho_{12}$

$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\text{anom}}$$

where

$$\sigma_{12,\text{anom}} = \frac{e^2 b}{2\pi^2}$$

Summary

• Chiral shift is necessarily generated \((B \neq 0, \mu \neq 0)\)

• Radiative corrections to \(\langle \vec{j}_5 \rangle\) are nonzero

• Radiative corrections are associated with the “matter” singularities on the Fermi surface

• Nonperturbative physics complicates the infrared contribution

• Potential applications range from astrophysics to condensed matter physics