Radiative corrections to chiral separation effect

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Helicities of (ultra-relativistic) massless particles are (approximately) conserved.

Conservation of chiral charge is a property of massless Dirac theory (classically).

The symmetry is anomalous at quantum level.

Right-handed

Left-handed
Chiral magnetic effect

- Chiral charge is produced by topological QCD configurations

\[
\frac{d(N_R - N_L)}{dt} = -\frac{g^2 N_f}{16\pi^2} \int d^3x \ F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a
\]

- Random fluctuations with nonzero chirality in each event

\[N_R - N_L \neq 0 \implies \mu_5 \neq 0\]

- Driving electric current

\[\langle \tilde{j} \rangle = \frac{e^2 \tilde{B}}{2\pi^2} \mu_5\]
Heavy ion collisions

- Dipole pattern of electric currents (charge correlations) in heavy ion collisions

[Fukushima, Kharzeev, Warringa, Phys. Rev. D 78, 074033 (2008)]
Experimental evidence

[B. I. Abelev et al. [The STAR Collaboration], arXiv:0909.1739]
[B. I. Abelev et al. [STAR Collaboration], arXiv:0909.1717]
Chiral separation effect

• Electric current induced by axial chemical potential

\[ \langle j^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu \quad \text{(free theory!)} \]

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

• Exact result (is it?), which follows from chiral anomaly relation

• No radiative correction expected…
Possible implication

- Seed chemical potential ($\mu$) induces axial current
  \[
  \langle j^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu
  \]
- Leading to separation of chiral charges:
  $\mu_5 > 0$ (one side) \& $\mu_5 < 0$ (another side)
- In turn, chiral charges induce back-to-back electric currents through
  \[
  \langle j^3 \rangle_{\text{free}} = \frac{e^2B}{2\pi^2} \mu_5
  \]
Quadrupole CME

- Start from a small baryon density and $B \neq 0$

- Produce back-to-back electric currents

  [Gorbar, Miransky, Shovkovy, Phys. Rev. D 83, 085003 (2011)]
Motivation

• Any additional consequences of the CSE relation?

\[ \langle j_5^3 \rangle_{\text{free}} = -\frac{eB}{2\pi^2} \mu \]  
  (free theory!)

[Metlitski & Zhitnitsky, Phys. Rev. D 72, 045011 (2005)]

• Any dynamical parameter \( \Delta \) ("chiral shift") associated with this condensate?

\[ \mathcal{L} = \mathcal{L}_0 + \Delta \bar{\psi} \gamma^3 \gamma^5 \psi \]

• Note: \( \Delta=0 \) is not protected by any symmetry
Chiral shift in NJL model

- NJL model (local interaction)

- "Gap" equations:
  \[ \mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle \]
  ("effective" chemical potential)
  \[ m = m_0 - G_{\text{int}} \langle \bar{\psi} \psi \rangle \]
  (dynamical mass)
  \[ \Delta = -\frac{1}{2} G_{\text{int}} \langle j^3 \rangle \]
  (chiral shift parameter)
Solutions

- Magnetic catalysis solution (vacuum state):

\[ m^2 \simeq \frac{|eB|}{\pi} \exp \left( -\frac{4\pi^2}{G_{\text{int}}|eB|} \right) \]

\[ \Delta = 0 \quad & \quad \mu = \mu_0 \]

- State with a chiral shift (nonzero density):

\[ m = 0 \quad & \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2} \]

\[ \Delta = \frac{g s \perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2} \]
Chiral shift @ Fermi surface

- Chirality is $\approx$ well defined at Fermi surface ($|k^3| \gg m$)
- L-handed Fermi surface:
  
  $n = 0 : \quad k^3 = +\sqrt{\left(\mu - s_\perp \Delta\right)^2 - m^2}$
  
  $n > 0 : \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$
  
  $k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$

- R-handed Fermi surface:
  
  $n = 0 : \quad k^3 = -\sqrt{\left(\mu - s_\perp \Delta\right)^2 - m^2}$
  
  $n > 0 : \quad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$
  
  $k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$
Chiral shift vs. axial anomaly

• Does the chiral shift modify the axial anomaly relation?

• Using point splitting method, one derives

\[
\langle \partial_\mu j_5^\mu (u) \rangle = - \frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left( e^{-is_{\perp} \Delta \epsilon^3} + e^{is_{\perp} \Delta \epsilon^3} \right)
\]

\[
\rightarrow - \frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \quad \text{for} \quad \epsilon \to 0
\]


• Therefore, the chiral shift does not affect the conventional axial anomaly relation
Axial current

• Does the chiral shift give any contribution to the axial current?

• In the point splitting method, one has

\[
\langle j_5^\mu \rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\varepsilon^2} \delta^3_\mu \equiv \frac{\Lambda^2\Delta}{2\pi^2} \delta^3_\mu
\]


• This is consistent with the NJL calculations

• Since \( \Delta \sim g\mu eB/\Lambda^2 \), the correction to the axial current should be finite
Axial current in QED

- Lagrangian density

\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \left( i \gamma^\mu D_\mu + \mu \gamma^0 - m \right) \psi + \text{(counterterms)} \]

- Axial current

\[ \langle j_5^3 \rangle = -Z_2 \text{tr} \left[ \gamma^3 \gamma^5 G(x,x) \right] \]

- To leading order in coupling \( \alpha = e^2/(4\pi) \)

\[ G(x,y) = S(x,y) + i \int d^4 u d^4 v \ S(x,u) \Sigma(u,v) S(v,y) \]
Expansion in external field

- Use expansion of $S(x,y)$ in powers of $A_{\mu}^{\text{ext}}$
- To leading order in coupling,

\[
\langle j_5^3 \rangle_0 = A_{\mu}^{\text{ext}}
\]

- The radiative correction is

\[
\langle j_5^3 \rangle_\alpha = A_{\mu}^{\text{ext}} + A_{\mu}^{\text{ext}} + A_{\mu}^{\text{ext}}
\]
Alternative form of expansion

- Expand \( S(x,y) = e^{i\Phi(x,y)}\bar{S}(x - y) \) in field

\[
S(x,y) = \bar{S}^{(0)}(x - y) + \bar{S}^{(1)}(x - y) + i\Phi(x,y)\bar{S}^{(0)}(x - y)
\]

- Translation invariant part
- Schwinger phase

- The Schwinger phase (in Landau gauge)

\[
\Phi(x,y) = -\frac{eB}{2} (x_1 + y_1)(x_2 - y_2)
\]

- Note: the phase is not translation invariant
Translation invariant parts

- Fourier transforms

$$\overline{S}^{(0)}(k) = i \frac{(k_0 + \mu)\gamma^0 - \mathbf{k} \cdot \gamma + m}{\left((k_0 + \mu + i \epsilon \text{sign}(k_0))^2 - \mathbf{k}^2 - m^2\right)}$$

$$\overline{S}^{(1)}(k) = -\gamma^1 \gamma^2 eB \frac{(k_0 + \mu)\gamma^0 - k_3\gamma^3 + m}{\left[\left((k_0 + \mu + i \epsilon \text{sign}(k_0))^2 - \mathbf{k}^2 - m^2\right)^2\right]}$$

- Note the singularity near the Fermi surface...
Fermi surface singularity

- “Vacuum” + “matter” parts

\[
\frac{1}{\left[ \left( k_0 + \mu + i \varepsilon \text{sign}(k_0) \right)^2 - k^2 - m^2 \right]^n} \Rightarrow "\text{Vac.}" + "\text{Mat.}"
\]

where

"Vac." = \[
\frac{1}{\left[ \left( k_0 + \mu \right)^2 - k^2 - m^2 + i \varepsilon \right]^n}
\]

"Mat." = \[
\frac{2\pi i (-1)^{n-1}}{(n-1)!} \theta(|\mu| - |k_0|) \theta(-k_0 \mu) \delta^{(n-1)} \left[ \left( k_0 + \mu \right)^2 - k^2 - m^2 \right]
\]
Axial current ($0^{th}$ order)

- From definition

\[ \langle j_5^3 \rangle_0 = - \int \frac{d^4 k}{(2\pi)^4} \text{tr} \left[ \gamma^3 \gamma^5 S^{(1)}(k) \right] \]

- After integrating over energy

\[ \langle j_5^3 \rangle_0 = - \frac{eB \text{sign}(\mu)}{4\pi^3} \int d^3 k \delta(\mu^2 - k^2 - m^2) \]

and finally

\[ \langle j_5^3 \rangle_0 = - \frac{eB \text{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2} \]

- Note the role of the Fermi surface (!)
Conventional wisdom

- Only the lowest (n=0) Landau level contributes

\[ \langle j^3 \rangle_0 = \frac{eB}{4\pi^2} \int d k_3 \left[ \theta \left( -\mu - \sqrt{k_3^2 + m^2} \right) - \theta \left( \mu - \sqrt{k_3^2 + m^2} \right) \right] \]

giving same answer

\[ \langle j^3 \rangle_0 = -\frac{eB \text{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2} \]

- There are no contributions from higher Landau levels (n≥1)

- There is a connection with the index theorem
Two facets

• Two ways to look at the same result

\[ B \rightarrow 0 \]

\[ B \neq 0 \]
Radiative correction

• Original two-loop expression

\[
\langle j_5^3 \rangle_\alpha = 32\pi \alpha e B \int \frac{d^4p \, d^4k}{(2\pi)^8} \frac{1}{(P - K)^2_\Lambda} \left[ \frac{(k_0 + \mu)[3(p_0 + \mu)^2 + p^2 + m^2] - 4(p_0 + \mu)(p \cdot k + 2m^2)}{(p^2 - m^2)^3(K^2 - m^2)} - \frac{(k_0 + \mu)[3(p_0 + \mu)^2 - p^2 + 3m^2] - 2(p_0 + \mu)(p \cdot k)}{3(p^2 - m^2)^2(K^2 - m^2)^2} \right] + \langle j_5^3 \rangle_{ct}.
\]

• After integration by parts

\[
\langle j_5^3 \rangle_\alpha = 64i\pi^2 \alpha e B \int \frac{d^4p \, d^4k}{(2\pi)^8} \left[ \frac{(k_0 + \mu)(p_0 + \mu) - p \cdot k - 2m^2}{(P - K)^2_\Lambda(K^2 - m^2)} \delta' \left[ \mu^2 - m^2 - p^2 \right] \delta(p_0) + \frac{3(p_0 + \mu)^2 - 3(k_0 + \mu)(p_0 + \mu) + p^2 - p \cdot k + 3m^2}{3(P - K)^2_\Lambda(P^2 - m^2)^2} \delta \left( \mu^2 - m^2 - k^2 \right) \delta(k_0) \right] + \langle j_5^3 \rangle_{ct}.
\]
Result \( (m \ll \mu) \)

- **Loop contribution**

\[
f_1 + f_2 + f_3 = \frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{\Lambda}{2\mu} + \frac{11}{12} \right) + \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{\Lambda}{2^{3/2} \mu} + \frac{1}{6} \right)
\]

- **Counterterm**

\[
\langle j^3_5 \rangle_{ct} = -\frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{\Lambda}{m} + \ln \frac{m^2}{m^2} + \frac{9}{4} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{\Lambda}{m^2} - \frac{3}{4} \right)
\]

- **Final result**

\[
\langle j^3_5 \rangle = -\frac{\alpha eB \mu}{2\pi^3} \left( \ln \frac{2\mu}{m} + \ln \frac{m^2}{m} + \frac{4}{3} \right) - \frac{\alpha eB m^2}{2\pi^3 \mu} \left( \ln \frac{2^{3/2} \mu}{m^2} - \frac{11}{12} \right)
\]
Sign of nonperturbative physics

- Unphysical dependence on photon mass

\[ \langle j_5^3 \rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3} \left( \ln \frac{2\mu}{m} + \ln \frac{m_\gamma^2}{m^2} + \frac{4}{3} \right) - \frac{\alpha eBm^2}{2\pi^3\mu} \left( \ln \frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12} \right) \]

- Infrared physics with

\[ m_\gamma \leq |k_0|, |k_3| \leq \sqrt{|eB|} \]

not captured properly

- Note: similar problem exists in calculation of Lamb shift
Nonperturbative effects (?)

• Perpendicular momenta cannot be defined with accuracy better than

\[ |\Delta k_\perp|_{\text{min}} \sim \sqrt{|eB|} \]

(In contrast to the tacit assumption in using expansion in powers of $B$-field)

• Screening effects provide a natural infrared regulator

\[ m_\gamma \Rightarrow \sqrt{\alpha \mu} \]

(Formally, this goes beyond the leading order in coupling)
Nonperturbative result (?

- Conjectured nonperturbative modification

(1) If non-conservation of momentum dominates

\[ \langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3} \left( \ln \frac{\mu |eB|}{m^3} + O(1) \right) - \frac{\alpha e B m^2}{2\pi^3 \mu} \left( \ln \frac{\mu}{\sqrt{|eB|}} + O(1) \right) \]

(2) If photon screening is more important

\[ \langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3} \left( \ln \frac{\alpha \mu^3}{m^3} + O(1) \right) - \frac{\alpha e B m^2}{2\pi^3 \mu} \left( \ln \frac{1}{\sqrt{\alpha}} + O(1) \right) \]
Summary (1)

• Weak $B$-field limit: new interpretation of the topological contribution to CSE relation

• Radiative corrections are nonzero

• Radiative corrections vanish without “matter” part with singularity on Fermi surface

• Nonperturbative physics complicates the infrared contribution

• With logarithmic accuracy, the result can be conjectured
Self-energy at $B \neq 0$

- **Self-energy**

\[ \Sigma(x,y) = -4i \pi \gamma^\mu S(x,y) \gamma^\nu D_{\mu\nu}(x-y) \]

- **General structure**

\[ \Sigma(x,y) = \exp(i \Phi(x,y)) \overline{\Sigma}(x-y) \]

- **Translation invariant part:**

\[ \overline{\Sigma}(p) = -4i \pi \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \overline{S}(k) \gamma^\nu D_{\mu\nu}(k-p) \]
Contribution linear in $B$

$$\overline{\Sigma}^{(1)}(p) = -4i\pi \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu \overline{S}^{(1)}(k) \gamma^\nu D_{\mu\nu}(k - p)$$

• The result has the form

$$\overline{\Sigma}^{(1)}(p) = \gamma^3 \gamma^5 \Delta + \gamma^0 \gamma^5 \mu_5(p)$$

where

$$\Delta \approx \frac{\alpha e B \mu}{\pi m^2} \left( \ln \frac{m^2}{2\mu(|p| - p_F)} - 1 \right)$$

$$\mu_5(p) \approx -\frac{\alpha e B \mu}{\pi m^2} \frac{p_3}{p_F} \left( \ln \frac{m^2}{2\mu(|p| - p_F)} - 1 \right)$$
Dispersion relations

• Let us use the condition

$$\text{Det} \left[ i \, S^{-1}(p) + \Sigma^{(1)}(p) \right] = 0$$
L/R-Fermi surface shift

\[ \frac{\langle p_3^{(0)} \rangle}{\mu} \]

\[ \begin{align*}
\text{L-handed} & \quad \text{R-handed}
\end{align*} \]

\[ p_\perp/\mu \]

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Summary (2)

• Chiral shift is generated in magnetized matter (evidence from renormalizable model now)

• The magnitude of chiral shift scales as

\[ \Delta \propto \frac{\alpha eB \mu}{m^2 \ln \alpha} \]

• Chiral shift induces a chiral asymmetry at the Fermi surface

• Chiral shift contributes to the axial current