Magnetic catalysis and chiral shift in dense matter

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+ work in progress

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**Dense relativistic matter**

- **Dense relativistic matter is common inside compact stars**
  - Electrons in white dwarfs
    \[ T \ll m \ll \mu \quad (i.e., \, T \ll 1 \text{ keV} \, \& \, \mu \approx 1 \text{ MeV}) \]
  - Neutrons of nuclear matter
    \[ T \ll m \ll \mu \quad (i.e., \, T \ll 10 \text{ MeV} \, \& \, \mu \approx 1 \text{ GeV}) \]
  - Electrons inside stellar nuclear matter
    \[ m \ll T \ll \mu \quad (i.e., \, T \ll 10 \text{ MeV} \, \& \, \mu \approx 100 \text{ MeV}) \]
  - Dense quark matter in stellar cores (if formed)
Dense relativistic matter

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General idea

- Topological current in relativistic matter in a magnetic field (3+1 dimensions)
  \[
  \langle \bar{\psi} \gamma^3 \gamma^5 \psi \rangle = \frac{eB}{2\pi^2} \mu 
  \]
  (free theory!)

  [Metlitski, Zhitnitsky, PRD 72, 045011 (2005)]

- Should there be a dynamical “mass” \( \Delta \), associated with this condensate?

- Note: \( \Delta = 0 \) is not protected by any symmetry
Lesson from graphene

- Dynamics of Quantum Hall Effect in graphene (≈ QED in 2+1 dimensions)
  - Parity and time-reversal odd Dirac mass

\[ \Delta \sim \langle \bar{\Psi} \gamma^3 \gamma^5 \Psi \rangle \]

[Gorbar, Gusynin, Miransky, I.S., PRB 78, 085437 (2008)]

\[ \Delta \text{ describes the } 0^{th} \text{ plateau in Quantum Hall effect in graphene} \]

[Δ = 1
\[ \Delta = -1 \]
\[ \Delta = 0 \]

Zhang et al., PRL 96, 136806 (2006)
Model

- Lagrangian density:

\[ \mathcal{L} = \bar{\psi} \left( i D_\nu + \mu_0 \delta^0_\nu \right) \gamma^\nu \psi + \frac{G_{\text{int}}}{2} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma^5 \psi)^2 \right] \]

- The dimensionless coupling is

\[ g \equiv \frac{G_{\text{int}} \Lambda^2}{(4\pi^2)} \ll 1 \]

- Magnetic field is inside

where

\[ A_\nu = x B \delta^2_\nu \]

(Landau gauge)
Approximation

- Gap equation in mean-field approximation:

\[
G^{-1}(u, u') = S^{-1}(u, u') - iG_{\text{int}} \left\{ G(u, u) - \gamma^5 G(u, u) \gamma^5 \right. \\
- \left. \text{tr}[G(u, u)] + \gamma^5 \text{tr}[\gamma^5 G(u, u)] \right\} \delta^4(u - u')
\]

where

\[
iG^{-1}(u, u') = \left[ (i\partial_t + \mu)\gamma^0 - (\pi \cdot \gamma) - \pi^3 \gamma^3 
+ i\bar{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m \right] \delta^4(u - u')
\]

and

\[
iS^{-1}(u, u') = \left[ (i\partial_t + \mu_0)\gamma^0 - (\pi \cdot \gamma) - \pi^3 \gamma^3 \right] \delta^4(u - u')
\]
Vacuum state

- Magnetic catalysis (spontaneous generation of a nonzero Dirac mass even at \( g \not= 1 \)):

\[
m_0^2 = \frac{1}{\pi l^2} \exp \left( -\frac{\Lambda^2 l^2}{g} \right)
\]

where

\[
l = 1/\sqrt{|eB|}
\]

(along with \( \mu = \mu_0 \))

[Gusynin, Miransky, I.S., PRL 73, 3499 (1994); PLB 349, 477 (1995)]

- The solution exists for \( \mu_0 < m_0 \), although it will be less stable than the normal state (\( m = 0 \)) already for

\[
\mu_0 \gtrsim m_0 \sqrt{2}
\]

[Clogston, PRL 29, 266 (1962)]
“Abnormal” normal ground state

- The gap equation allows another solution,
  \[ \mu \simeq \mu_0 \text{ and } \Delta \simeq g\mu_0 eB/\Lambda^2 \]

- This solution is almost independent of temperature when \( T \gtrapprox \mu \)

- This is the normal ground state since its symmetry is same as in the Lagrangian

- Besides, there is no trivial solution \( \Delta=0 \)
Change of ground state

- The free energy in the state with $m \neq 0$ (broken symmetry)

$$\Omega_m \simeq - \frac{m_0^2}{2(2\pi l)^2} \left( 1 + (m_0 l)^2 \ln |\Lambda l| \right)$$

- The free energy in the normal state, $\Delta \neq 0$

$$\Omega_\Delta \simeq - \frac{\mu_0^2}{(2\pi l)^2} \left( 1 - g \frac{|eB|}{\Lambda^2} \right)$$

- So, indeed symmetry is restored for $\mu > \mu_c$

$$\mu_c \simeq \frac{m_0}{\sqrt{2}}$$
Physical meaning of $\Delta$

- The dispersion relation of quasiparticles:

$$\omega_{n,\sigma} = -\mu \pm \sqrt{[k_3 + \sigma \Delta]^2 + 2n|eB|}$$

where $\mathbb{W} = \pm 1$ is the chirality

- Longitudinal momenta of opposite chirality fermions are shifted, i.e., $k_3 \mathbb{W} k_3 \pm \Delta$

- All Landau levels $(n \mathbb{W} 0)$ are affected by $\Delta$
Magnetic catalysis at $B_0 = 0$
T=0 results
T≠0 results

- These are smoothed versions of the T=0 results
- The dependence $\Delta$ versus $\mu_0/m_{\text{dyn}}$ at $T=0$ is similar to $\Delta$ versus $\mu_0$ (not shown) at $T≠0$ is similar to $\Delta$ versus $\mu_0$ (shown)
Induced axial current

- The axial current in the ground state is

\[
\langle \bar{\psi} \gamma^3 \gamma^5 \psi \rangle = \left( \frac{eB}{2\pi^2 \mu} \right) \Delta - \left( \frac{|eB|}{\pi^2} \right) \Delta \sum_{n=1}^{\infty} \kappa(\sqrt{2n|eB|}, \Lambda)
\]

- In addition to the topological contribution, there are dynamical ones \( \Delta \)

- An equivalent result is also obtained in the Pauli-Villars regularization

- **Note:** on the solution to the gap equation:

\[
\langle j_5^3(u) \rangle = \frac{2\Delta}{G_{int}} = \frac{\Delta}{2\pi^2} \frac{\Lambda^2}{g}
\]
Potential implications

- Physical properties to be affected
  - transport
  - emission
    (must be sensitive to anisotropy and/or CP violation)

- Specific physical systems
  - Compact stars
    - Quark stars (quarks)
    - Hybrid stars (quarks, electrons)
    - Neutron stars (electrons)
    - White dwarfs (electrons)
  - Heavy ion collisions [Kharzeev & Zhitnitsky, NPA 797, 67(2007),
    Kharzeev, McLerran & Warringa, NPA 803, 227 (2008), …]
Pulsar kicks

- The dynamical chiral shift parameter is driven by chemical potential ($T \mu$)

$$\Delta \simeq g \mu_0 eB / \Lambda^2$$

and is almost independent of temperature

- This creates an anisotropy in the distribution of left-handed quarks/electrons

- The anisotropy is transferred to left-handed neutrinos by elastic scattering

- Pulsar gets a kick when neutrinos escape
Supernova explosions

- Because of the robustness of the axial currents at finite temperatures, even supernova explosions may be affected.

- A small early-time neutrino asymmetry may facilitate explosions and give a kick at the same time, e.g., see [Fryer & Kusenko, Astrophys. J. Supp. 163, 335 (2006)]
Summary

- $\mu < \mu_c$: Chiral symmetry is broken in the ground state (magnetic catalysis)

- $\mu > \mu_c$: Normal ground state of dense relativistic matter in a magnetic field is characterized by
  - Chiral shift parameter (may have dramatic implications for stars)
  - Axial current along the field (physical effects are not obvious)
  - No solution with vanishing $\Delta$ exists
Outlook

- Calculation of neutrino emission/diffusion in the state with a chiral shift parameter (work in progress)
- Transport properties of the normal state with nonzero chiral shift parameter
- The fate of the induced axial current in the renormalized models (work in progress)
- Modification of the chiral magnetic effect due to “vector-like” $\Delta$ in heavy ion collisions

[Fukushima, Kharzeev & Warringa, PRD 78, 074033 (2008)]
Thank you