Graphene: Symmetry breaking in the carbon Flatland*

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What is graphene?

• It is a single atomic layer of graphite, see [Novoselov et al., Science 306, 666 (2004)]

2D crystal with hexagonal lattice of carbon atoms
Lattice in coordinate/reciprocal space

- Two carbon atoms per primitive cell
- Translation vectors

\[ \mathbf{a}_1 = a \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right), \quad \mathbf{a}_2 = a \left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) \]

where \( a \) is the lattice constant

- Reciprocal lattice vectors

\[ \mathbf{b}_1 = \frac{2\pi}{a(1, 1/\sqrt{3})}, \quad \mathbf{b}_2 = \frac{2\pi}{a(1, -1/\sqrt{3})} \]
Tight binding model

• There are strong covalent sigma-bonds between nearest neighbors

• Hamiltonian

\[ H = -t \sum_{n, \delta_i, \sigma} \left[ a_{n, \sigma}^\dagger \exp \left( \frac{ie}{\hbar c} \delta_i A \right) b_{n+\delta, \sigma} + c.c. \right] \]

where \( a_{n,\sigma} \) and \( b_{n+\delta,\sigma} \) are the annihilation operators of electrons with spin \( \sigma = \uparrow, \downarrow \)

• The nearest neighbor vectors are

\[ \delta_1 = (a_1 - a_2)/3, \quad \delta_2 = a_1/3 + 2a_2/3, \]
\[ \delta_3 = -\delta_1 - \delta_2 = -2a_1/3 - a_2/3 \]
Low energy Dirac fermions

\[ \mathcal{L} = \sum_{\sigma = \pm 1} \bar{\Psi}_\sigma(t, \mathbf{r}) \left[ i \gamma^0 (\hbar \partial_t - i \mu_\sigma) + i \hbar v_F \gamma^1 D_x + i \hbar v_F \gamma^2 D_y \right] \Psi_\sigma(t, \mathbf{r}) \]

P. R. Wallace, Phys Rev 71, 622 (1947)
Quantum Hall effect in graphene

\[ E_N = \pm v_F \sqrt{2e\hbar BN} \]

\[ \sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{\hbar} \left( n + \frac{1}{2} \right) \]

Quantum Hall Effect at large $B$

There are new plateaus at
\[ \nu = 0, \quad \nu = \mp 1, \quad \nu = \mp 4 \]
i.e., the degeneracy of some Landau levels is lifted

See also
Abanin et al., PRL 98, 196806 (2007)
Jiang et al., PRL 99, 106802 (2007)
Checkelsky et al., PRL 100, 206801 (2008)
Magnetic catalysis (MC) scenario

Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

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It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

\[ E_n = \sqrt{2n|eB|} \Rightarrow E_n = \sqrt{2n|eB|} + \Delta_0^2 \]

where \[ \Delta_0 \sim \sqrt{|eB|} \Rightarrow \nu=0 \]

First proposed for graphene in

D.V. Khveshchenko, PRL 87, 206401 (2001); ibid. 87, 246802 (2001)
E.V. Gorbar, V.P. Gusynin, V.A. Miransky, and I.A. Shovkovy, PRB 66, 045108 (2002).
Quantum Hall Ferromagnetism (QHF)

Arovas, Karlhelde, & Lilliehook, PRB 59, 13147 (1999)
Ezawa & Hasebe, PRB 65, 075311 (2002)

• Spin/valley degeneracy of the half-filled Landau level is lifted by the exchange (repulsive Coulomb) interaction
• This is similar to the **Hund’s Rule(s)** in atomic physics
• In the lowest energy state, the coordinate part of the wave function is *antisymmetric* (with the electrons being as far apart as possible)
  i.e., it is *symmetric* in the spin/valley indices
• This is nothing else but ferromagnetism
Model Hamiltonian

\[ H = H_0 + H_C + \int d^2 r \left[ \mu_B B \Psi^\dagger \sigma^3 \Psi - \mu_0 \Psi^\dagger \Psi \right] \]

where

\[ H_0 = v_F \int d^2 r \, \overline{\Psi} \left( \gamma^1 \pi_x + \gamma^2 \pi_y \right) \Psi, \]

is the Dirac Hamiltonian, and

\[ H_C = \frac{1}{2} \int d^2 r \, d^2 r' \Psi^\dagger (r) \Psi (r) U_C (r - r') \Psi^\dagger (r') \Psi (r') \]

is the Coulomb interaction term.

Note that

\[ \Psi^T_s = (\psi_{KAs}, \psi_{KBS}, \psi_{K'Bs}, \psi_{K'As}) \]

\[ v_F \approx 10^6 \text{ m/s} \]
Symmetry

• The Hamiltonian \( H = H_0 + H_C \) possesses “flavor” \( U(4) \) symmetry

• 16 generators read \((\text{spin } \otimes \text{ pseudospin})\)

\[
\begin{align*}
\frac{\sigma^\alpha}{2} \otimes I_4, & \quad \frac{\sigma^\alpha}{2i} \otimes \gamma^3, \\
\frac{\sigma^\alpha}{2} \otimes \gamma^5, & \quad \text{and} \quad \frac{\sigma^\alpha}{2} \otimes \gamma^3 \gamma^5.
\end{align*}
\]

• The Zeeman term breaks \( U(4) \) down to \( U(2)_+ \times U(2)_- \).

• Dirac mass breaks \( U(2)_s \) down to \( U(1)_s \).
Energy scales in the problem

- **Landau energy scale**
  $$\epsilon_B \equiv \sqrt{2\hbar|eB_\perp|v_F^2/c} \approx 424\sqrt{|B_\perp[T]|}\text{ K}$$

- **Zeeman energy**
  $$Z \approx \mu_B B = 0.67B[T]\text{ K}$$

- **Dynamical mass scales** ($Z \ll A \leq M \ll \epsilon_B$)
  $$A \equiv \frac{G_{\text{int}}|eB_\perp|}{8\pi\hbar c} = \frac{\sqrt{\pi}\lambda\epsilon_B^2}{4\Lambda}$$

- In our calculations,
  $$M = 4.84 \times 10^{-2}\epsilon_B \text{ and } A = 3.90 \times 10^{-2}\epsilon_B$$
Full propagator

- We use the following general ansatz:

\[ iG_s = \left[ (i\hbar\partial_t + \mu_s + \tilde{\mu}_s \gamma^3 \gamma^5)\gamma^0 - v_F (\mathbf{\pi} \cdot \gamma) - \tilde{\Delta}_s + \Delta_s \gamma^3 \gamma^5 \right]^{-1} \]

- Physical meaning of the order parameters

\[ \Delta_s : \quad \bar{\Psi} \gamma^3 \gamma^5 P_s \Psi = \psi^\dagger_{KAs} \psi_{KA_s} - \psi^\dagger_{K'A_s} \psi_{K'A_s} - \psi^\dagger_{KBs} \psi_{KB_s} + \psi^\dagger_{K'Bs} \psi_{K'B_s} \]

\[ \tilde{\Delta}_s : \quad \bar{\Psi} P_s \Psi = \psi^\dagger_{KAs} \psi_{KA_s} + \psi^\dagger_{K'A_s} \psi_{K'A_s} - \psi^\dagger_{KBs} \psi_{KB_s} - \psi^\dagger_{K'Bs} \psi_{K'B_s} \]

\[ \mu_3 : \quad \Psi^\dagger \sigma^3 \Psi = \frac{1}{2} \sum_{\kappa=K,K'} \sum_{a=A,B} \left( \psi^\dagger_{\kappa a+} \psi_{\kappa a+} - \psi^\dagger_{\kappa a-} \psi_{\kappa a-} \right) \]

\[ \tilde{\mu}_s : \quad \bar{\Psi} \gamma^3 \gamma^5 P_s \Psi = \psi^\dagger_{KAs} \psi_{KA_s} - \psi^\dagger_{K'A_s} \psi_{K'A_s} + \psi^\dagger_{KBs} \psi_{KB_s} - \psi^\dagger_{K'Bs} \psi_{K'B_s} \]
Schwinger Dyson equation

- Hartree-Fock (mean field) approximation:

(a) Coulomb interaction

\[
S = S + S + S + S
\]

(b) Contact interaction

\[
S = S + S + S + S
\]
Three types of solutions

i. \( S \) (singlet with respect to \( U(2)_s \) where \( s=\uparrow,\downarrow \))
   - Order parameters: \( \mu_3 \) and/or \( \Delta_s \)
   - Symmetry: \( U(2)_+ \times U(2)_- \)

ii. \( T \) (triplet with respect to \( U(2)_s \))
   - Order parameters: \( \tilde{\mu}_s \) and/or \( \tilde{\Delta}_s \)
   - Symmetry: \( U(1)_+ \times U(1)_- \)

iii. \( H \) (hybrid, i.e., singlet + triplet)
   - Order parameters: mixture of \( S \) and \( T \) types
   - Symmetry: \( U(2)_+ \times U(1)_- \) or \( U(1)_+ \times U(2)_- \)
Solutions at LLL \((\mu_0 \ll \epsilon_B)\)

\[
\frac{\Omega}{L^3} \quad \mu_0/L
\]

T=1 K

- S
- T
- H1
- H2
- TR

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Singlet solution vs. $T$ ($\nu=0$ QHE state)

\[ \tilde{\Delta}_+ = \tilde{\mu}_+ = 0, \quad \mu_+ = \bar{\mu}_+ - A, \quad \Delta_+ = s_\perp M, \]
\[ \tilde{\Delta}_- = \tilde{\mu}_- = 0, \quad \mu_- = \bar{\mu}_- + A, \quad \Delta_- = -s_\perp M. \]
Singlet solution ($\nu=0$ & $2$ QHE states)
Hybrid solution ($\nu=1$ QHE state)

$\tilde{\Delta}_+ = M, \quad \tilde{\mu}_+ = A s_\perp, \quad \mu_+ = \bar{\mu}_+ - 4A, \quad \Delta_+ = 0,$

$\tilde{\Delta}_- = \tilde{\mu}_- = 0, \quad \mu_- = \bar{\mu}_- - 3A, \quad \Delta_- = -s_\perp M.$

![Graphs showing hybrid solution behavior for different temperatures](image1.png)

![Graphs showing hybrid solution behavior for different temperatures](image2.png)
Hybrid solutions at 1\textsuperscript{st} Landau level

\begin{align*}
T &= 1 \text{ K} \\
\cdot \mu_{cr1}/L &= 1.79 \\
\cdot \mu_{cr2}/L &= 2.12 \\
\cdot \mu_{cr3}/L &= 2.26 \\
\cdot \mu_{cr4}/L &= 2.58
\end{align*}
Phase diagram
Theory vs. experiment (1)

- Theory predicts all “new” plateaus observed in a strong magnetic field (i.e., $\nu=0$, $\nu=\pm 1$, $\nu=\pm 4$)

- The plateaus $\nu=\pm 3$, $\nu=\pm 5$, which are not observed yet, are also predicted

- This might be in a qualitative agreement with a suggested large width of higher Landau levels [Giesbers et al., PRL 99, 206803 (2007)]
Theory vs. experiment (2)

First try with a “reasonable” set of parameters

\[ \sigma [e^2/h] \]

\[ T \text{ [Kelvin]} \]

Experiment ⇒

Checkelsky, Li, Ong,

PRL 100, 206801 (2008)

\[ \sigma_{xx}^0 (e^2/h) \]

\[ T \text{ (K)} \]

7 6 T
8
9
10
11
12
13
14

K7 (a)
Summary

- A rich phase diagram in the $T-\mu$ plane is proposed.
- Both MC and QHF are responsible for dynamical symmetry breaking and lifting the degeneracy of Landau levels in graphene.
- Qualitative agreement with experiment is evident, but details remain to be worked out.