Dense baryon matter: progress and difficulties

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Igor A. Shovkovy  QCD at Finite Density  ECT*, Trento, March 21-25, 2006
I. Introduction into color superconductivity

- Phase diagram of baryonic matter
- Color superconductivity
  - 2SC (2 quark flavors)
  - CFL (3 quark flavors)
  - Spin-1 color superconductivity (1 quark flavor)

II. Cooper pairing under stress

- Neutrality vs. color superconductivity
- Unconventional pairing in color superconductors
- Gapless, crystalline and other phases
- Current status
- Summary
Compact (neutron) stars

- Radius: 
  \[ R \approx 10 \text{ km} \]

- Mass: 
  \[ 1.25M_\odot \lesssim M \lesssim 2M_\odot \]

- Core temperature: 
  \[ 10 \text{ keV} \lesssim T \lesssim 10 \text{ MeV} \]

- Surface magnetic field: 
  \[ 10^8 \text{ G} \lesssim B \lesssim 10^{14} \text{ G} \]

- Rotational period: 
  \[ 1.6 \text{ ms} \lesssim P \lesssim 12 \text{ s} \]

Central densities in stars should be rather high: \[ \rho_c \gtrsim 5\rho_0 \]
Possible phases of matter inside stars

Hydrogen/Heatmosphere

R ~ 10 km

n,p,e, µ

neutron star with pion condensate

quark−hybrid star

hyperon star

traditional neutron star

n superfluid

neutron star with pion condensate

crust

strange star

nucleon star

Hydrogen/He atmosphere

R ~ 10 km

[figure from F. Weber, astro-ph/0407155 (modified)]
Very dense baryonic matter

Baryons at high density → quark matter

- Asymptotic freedom: \( \alpha_s(\mu) \ll 1 \)
  \( \mu \gg \Lambda_{QCD} \) [Gross&Wilczek; Politzer,’73]

⇒ Weakly interacting regime
  [Collins&Perry,’75]

Note: realistic densities in stars may not be sufficiently large:

\[ \rho \lesssim 10 \rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3} \Rightarrow \mu \lesssim 500 \text{ MeV} \]
Two complimentary approaches

(i) QCD (from first principles):

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_f^\alpha \left( i \gamma^\mu \partial_\mu + \gamma^0 \mu_f + g T^a_{\alpha\beta} \gamma^\mu A^a_\mu - m_f \right) \psi_f^\beta - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} \]

- predictions are reliable when \( \mu \gg \Lambda_{\text{QCD}} \)

(ii) Phenomenological (e.g., NJL-type) models fitted to reproduce basic properties of vacuum QCD and/or nuclear matter, e.g.,

\[ \mathcal{L}_{\text{NJL}} = \bar{\psi}_f^\alpha \left( i \gamma^\mu \partial_\mu + \gamma^0 \mu_f - m_f \right) \psi_f^\beta + \frac{g^2}{2} (\bar{\psi} \gamma^\mu T^a \psi)(\bar{\psi} \gamma_\mu T^a \psi) \]

- may work only when \( \rho \lesssim \rho_0 \)

Note: densities of interest: \( 3 \rho_0 \lesssim \rho \lesssim 10 \rho_0 \)
Gluon propagator in dense medium:

\[ D_{\mu\nu}^{-1}(k) = D_{0,\mu\nu}^{-1}(k) - \Pi_{\mu\nu}(k), \quad \text{i.e.,} \quad \frac{k}{\not{k}} = \frac{k}{\not{k}} + \Gamma \]

Electrical Debye screening and magnetic dynamical screening

[Son, hep-ph/9812287]:

\[ iD_{\mu\nu}(k_4, |\vec{k}|) \simeq - \frac{O^{(el)}_{\mu\nu}}{k_4^2 + |\vec{k}|^2 + 2M_D^2} - \frac{|\vec{k}|O^{(mag)}_{\mu\nu}}{|\vec{k}|^3 + \pi M_D^2 |k_4|/2}, \]

where \( M_D^2 = \alpha_s N_f \mu^2 / \pi \) is the Debye mass

Magnetic interaction is long-ranged in (near-)static limit, \( k_4 \ll |\vec{k}| \ll \mu \)
QCD: gap equation & solution

Gap equation:

\[
\Delta_{p}^{\pm} = \frac{4\pi\alpha_s}{3} \int \frac{d^4k}{(2\pi)^4} \left( \frac{\Delta_{k}^{-} \text{Tr} \left( \gamma_{\mu} \Lambda_{k}^{+} \gamma_{\nu} \Lambda_{p}^{\pm} \right)}{k_0^2 - (|k| - \mu)^2 - |\Delta_{k}^{-}|^2} + \frac{\Delta_{k}^{+} \text{Tr} \left( \gamma_{\mu} \Lambda_{k}^{-} \gamma_{\nu} \Lambda_{p}^{\pm} \right)}{k_0^2 - (|k| + \mu)^2 - |\Delta_{k}^{+}|^2} \right) D_{k-p}^{\mu\nu}
\]

Approximate solution for \( \Delta_0 = |\Delta_{k}^{-}|_{k \approx \mu} \)

\[
\Delta_0 \simeq \lambda \mu \quad \exp \left( -\frac{3\pi^{3/2}}{2^{3/2}\sqrt{\alpha_s}} \right) \quad \text{long-range magnetic gluons}
\]

where

\[
\lambda = \frac{2(4\pi)^{3/2}}{\alpha_s^{5/2}} \times \exp \left( -\frac{4 + \pi^2}{8} \right) \times \text{(higher order corrections)}
\]

electric gluons \quad \text{quark self-energy corrections}

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$N_f = 2$ color superconductivity (2SC)

Simplest case, **2SC phase** [Barrois,’78; Bailin&Love,’84]

- $N_f = 2$: “up” and “down”

- $N_c = 3$: “red”, “green” and “blue”

Diquark condensate:

$$\phi_3 = \left\langle \left( \bar{\Psi}^{\alpha} \right)_i \epsilon^{ij} \epsilon^{\alpha\beta} \gamma^5 \Psi^{\beta}_j \right\rangle \neq 0$$

(Pauli principle)

Note:

- Same colors and same flavors do not pair (spin-0 channel)
- Only two colors of quarks participate in Cooper pairing

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Symmetries of 2SC state

• Diquark condensate:

\[ \langle (\bar{\Psi}^C)^{\alpha}_{\ i} \gamma_5 \Psi^\beta_{\ j} \rangle \sim \varepsilon^{a\beta} \epsilon_{ij} \Delta \]


✓ baryon number \( U(1)_B \rightarrow \tilde{U}(1)_B \) with \( \tilde{B} = B - \frac{2}{\sqrt{3}} T_8 \)

(quark matter is not superfluid)

✓ gauge symmetry \( U(1)_{em} \rightarrow \tilde{U}(1)_{em} \) with \( \tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8 \)

(there is no Meissner effect)

– chiral \( SU(2)_L \times SU(2)_R \) — intact

– approximate axial \( U(1)_A \) is broken → 1 pseudo-NG boson

– color gauge symmetry \( SU(3)_c \rightarrow SU(2)_c \) (Higgs mechanism)
\( N_f = 3 \) color superconductivity

Diquark condensate:

\[
\left\langle \left( \bar{\psi}^C \right)_i \epsilon^{ijk} \epsilon_{abc} C \gamma^5 \psi^b_j \right\rangle \sim \delta^c_k
\]

or, in terms of chiral fields,

\[
\left\langle \psi^{a,\alpha}_{L,i} \epsilon^{ijk} \epsilon_{abc} \epsilon_{\alpha\beta} \psi^{b,\beta}_{L,j} \right\rangle = - \left\langle \psi^{a,\dot{\alpha}}_{R,i} \epsilon^{ijk} \epsilon_{abc} \epsilon_{\dot{\alpha}\dot{\beta}} \psi^{b,\dot{\beta}}_{R,j} \right\rangle \sim \delta^c_k
\]

Color-flavor locking:

[Alford, Rajagopal, & Wilczek, hep-ph/9804403]

\[
\langle LL \rangle \quad \text{is invariant under} \quad g_{L, \text{flavor}} \otimes g_{\text{color}} \quad \text{if} \quad g_{\text{color}} \equiv g_{L, \text{flavor}}^{-1}
\]

\[
\langle RR \rangle \quad \text{is invariant under} \quad g_{R, \text{flavor}} \otimes g_{\text{color}} \quad \text{if} \quad g_{\text{color}} \equiv g_{R, \text{flavor}}^{-1}
\]

i.e., residual global symmetry is \( SU(3)_{L+R} \)

As in vacuum, there appear 8 corresponding Nambu-Goldstone bosons:

\( \pi^0, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta \)
chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$

$\to 8$ (pseudo-)NG bosons, i.e., $\pi^0$, $\pi^\pm$, $K^\pm$, $K^0$, $\bar{K}^0$, $\eta$

(almost like in vacuum QCD)

baryon number $U(1)_B$ is broken $\to 1$ NG boson ($\phi$)

(quark matter is superfluid)

gauge symmetry $U(1)_{em} \to \tilde{U}(1)_{em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$

(there is no Meissner effect)

- approximate axial $U(1)_A$ is broken $\to 1$ pseudo-NG boson ($\eta'$)

- color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism

$\to 8$ massive gluons
\( N_f = 1 \) color superconductivity

- Cooper pair:  \(|\bullet\bullet\rangle - |\bullet\bullet\rangle\rangle_3 \otimes \left|\uparrow\uparrow\right\rangle_{J=1}

- Diquark condensate:

\[
\langle (\bar{\Psi}^C)^{\alpha} \gamma_5 \Psi^\beta \rangle \approx \varepsilon^{\alpha\beta\gamma} \Delta_{c\delta} \left( \hat{k}^\delta \sin \theta + \gamma_\perp (\vec{k}) \cos \theta \right)
\]


- Many possibilities, e.g., see [Schmitt, nucl-th/0412033]:
  - Color-spin-locked phase: \( \Delta_{c\delta} = \delta_{c\delta} \rightarrow \text{largest pressure (?)} \)
  - Planar phase: \( \Delta_{c\delta} = \delta_{c\delta} - \delta_{c3} \delta_{33} \)
  - Polar phase: \( \Delta_{c\delta} = \delta_{c3} \delta_{33} \)
  - A-phase: \( \Delta_{c\delta} = \delta_{c3} (\delta_{31} + i\delta_{32}) \rightarrow \text{unusual neutrino emission} \)

- Many similarities with superfluidity in \(^3\text{He} \) . . .
Matter in the bulk of a star must be

(i) in $\beta$-equilibrium: $\mu_d = \mu_u + \mu_e$,
    i.e., the weak processes
    $$u + e^- \rightarrow d + \nu$$
    & $$d \rightarrow u + e^- + \bar{\nu}$$

    have equal rates;

(ii) electrically and color neutral:
    $$n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$$

If $n_Q \neq 0$, the Coulomb energy is

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left( \frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left( \frac{R}{1 \text{ km}} \right)^5$$

e.g., if $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2SC} \gg M_\odot c^2$
Unconventional Cooper pairing, $N_f = 2$

- The “best” 2SC phase appears when $n_d \approx n_u$

- Neutral matter appears when $n_d \approx 2n_u$

- Electrons, required in $\beta$ equilibrium, cannot help:

$$n_d \approx 2n_u \quad \text{where} \quad n_d = \frac{\mu_d^3}{\pi^2}, \quad n_u = \frac{\mu_u^3}{\pi^2}$$

i.e.,

$$\mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u \Rightarrow n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$$

Therefore, Cooper pairing is unavoidably distorted by the “mismatch” parameter:

$$\delta \mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$

What happens then?
Gapless 2SC phase

Competition: \( \delta \mu \) vs. \( \Delta_0 \) (where \( \Delta_0 \) is the gap at \( \delta \mu = 0 \))

The “winner” is determined by the diquark coupling strength


1. \( \delta \mu \gtrsim \Delta_0 \) — the mismatch does not allow Cooper pairing:
   normal phase is the ground state

2. \( \delta \mu \lesssim \frac{1}{2} \Delta_0 \) — coupling is strong enough to win over the mismatch:
   2SC is the ground state

3. \( \frac{1}{2} \Delta_0 \lesssim \delta \mu \lesssim \Delta_0 \) — regime of intermediate coupling strength:
   the ground state is the gapless 2SC phase

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Quasiparticle spectrum in g2SC phase

“Intermediate” coupling

The energy gaps in the quasiparticle spectra are

\[ 0 \quad \& \quad \Delta + \delta \mu \]
Chromomagnetic instability

Recent results for gluon screening masses

[Huang & Shovkovy, hep-ph/0407049]:

\[ \frac{m_D^2}{m_g^2} \]

\[ \frac{\Delta \mu}{m_D^2} \]

\[ \frac{m_M^2}{m_g^2} \]

\[ \Delta \mu \]

\[ \sqrt{2} \]

\[ m_M^2 < 0 \]

\[ A = 1, 2, 3 \] — red solid line

\[ A = 4, 5, 6, 7 \] — green long-dash line

\[ A = \tilde{8} \] — blue short-dash line

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- 4-7th: \[ p_0^2 = m^2 + v^2 p^2 \]
  \[ m^2 > 0 \quad \text{for} \quad \Delta > \sqrt{2}\delta\mu \]
  \[ m^2 < 0 \quad \text{for} \quad \Delta < \sqrt{2}\delta\mu \]

- 8th: \[ p_0^2 = v^2 p^2 \] with \( v^2 < 0 \)
  appearing only for \( \Delta < \delta\mu \)

Two types of tachyons \( \rightarrow \) two type of ground states

- 4-7th: \( \langle A^{(4)} \rangle = \text{const} \)

- 8th: \( \langle A^{(8)}(x) \rangle \neq \text{const} \)
  i.e., more than 1-wave LOFF
\( N_f = 2 + 1 \) color superconductivity, \( 0 < m_s < \infty \)

Fermi momentum of strange quarks is lowered:

\[
k^s_F \simeq \mu - \frac{m^2_s}{2\mu}
\]

Then, the ground state is defined by:

(?) only condensates of same flavor (spin-1 channel)

(?) only superconductivity of up and down quarks (2SC or g2SC)

(?) gapless CFL phase (\( \oplus \) yet unknown stabilization mechanism)

[Alford, Kouvaris & Rajagopal, hep-ph/0311286]

(?) crystalline pairing (nonzero momentum pairing, LOFF)

[Alford, Bowers & Rajagopal, hep-ph/0008208]

(?) P-wave kaon condensates

[Schafer, hep-ph/0508190]
Cooper pairs with nonzero momenta:

\[ z p \mu x p = \mu | q | \mu | q | q | \mu \]

\[ z u d p-q u p+q 2 \]

[Igor A. Shovkovy, QCD at Finite Density, ECT*, Trento, March 21-25, 2006]
P-wave kaon condensation

Baryon spectrum in CFL phase

[Kryjevski & Schafer, hep-ph/0407329]

Within the framework of effective theory:

P-wave meson condensation

See also

[Son & Stephanov, cond-mat/0507586] ⇒

[Kryjevski, hep-ph/0508180],

[Schafer, hep-ph/0508190]

⇒ No instabilities
Dense quark matter may be “modeled” in a tabletop experiment by studying trapped cold gases of fermionic atoms (e.g., $^{6}\text{Li}$ or $^{40}\text{K}$)

First experimental results:

Zwierlein et. al., cond-mat/0511197*
Partridge et. al., cond-mat/0511752
Summary

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative
- In this limit, QCD can be studied from first principles
- Under conditions in stars, Cooper pairing is unconventional
- There may exist many different phases in the QCD phase diagram as well as in stars
- Physics of stars and physics of matter around us might be closer related than one might naively expect . . .

Many problems remain:

(i) instabilities of gapless phases

(ii) inhomogeneous ground states

(iii) search for observables, etc.
Some reviews on color superconductivity

- K. Rajagopal and F. Wilczek, “The condensed matter physics of QCD”
  hep-ph/0011333

- M. Alford, “Color superconducting quark matter”


- D. H. Rischke, “The quark-gluon plasma in equilibrium”


- I. A. Shovkovy, “Two lectures on color superconductivity”