On recent progress in color superconductivity

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Matter at high density

Why should one be interested in studying dense matter?

(i) Dense matter exists in the Universe

(densities in stars \( \rho_c \gtrsim 5\rho_0 \))

(ii) Fundamental properties of QCD

(\( \mu \gtrsim \Lambda_{QCD} \): no lattice results)
Phases of matter inside compact stars

- Neutron star with pion condensate
- Quark-hybrid star
- Neutron superfluid
- Strange quark matter (u,d,s quarks)
- Traditional neutron star
- Hyperon star
- Nucleon star
- Hyperon star
- Quark-hybrid star
- Strange star
- Color-superconducting strange quark matter (u,d,s quarks)

[figure from F. Weber, astro-ph/0407155]

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Dense matter might be deconfined

- “Squeezing” baryonic matter hard should produce quark matter:

- Conjecture: quark matter may exist in stars
  [Ivanenko & Kurdyga, ’65], [Itoh ’70], [Collins & Perry, ’75]

- What is the ground state of quark matter?
- What is the effect of charge neutrality and $\beta$-equilibrium?
Ground state of dense quark matter

Educated guess:

(i) Quarks are fermions \( (s = \frac{1}{2}) \) \( \Rightarrow \) Fermi liquid (?)

(ii) Interaction is weak \( (\alpha_s \ll 1) \)

(cf., electron gas in metals/alloys)

Further refinement:

(i) Degenerate Fermi surface

(ii) Attractive interaction (?) \( \Rightarrow \) Cooper instability

(cf., the Cooper instability in superconducting metals/alloys)
**Color superconductivity in dense QCD**

Simplest case, 2SC phase [Barrios,’78; Bailin&Love,’84; Son,’99]

- $N_f = 2$: “up” and “down” quarks
- $N_c = 3$: “red”, “green” and “blue”
- $p_F^{up} = p_F^{down} = \mu$
- Quark-quark interaction:

\[
\begin{align*}
\langle u_p d_p \rangle & = - \langle u_q d_q \rangle \\
\frac{1}{3} & \Rightarrow \text{Repulsive}
\end{align*}
\]

Cooper instability $\rightarrow$ color superconductivity

\[
(\left| \bar{u} \downarrow \right> - \left| u \uparrow \right>)_3 \otimes (\left| \bar{d} \downarrow \right> - \left| d \uparrow \right>)_0 \otimes (|u,d\rangle - |d,u\rangle) \quad (\Leftrightarrow \text{Pauli principle})
\]
2SC ground state properties

- Wave function of a Cooper-pair:
  Pauli principle: \((|\bullet\bullet\rangle - |\bullet\bullet\rangle)\frac{3}{2} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_{1}\)

- Diquark condensate (spin-0 gap \(\sim 10 - 100\) MeV):
  \[ \langle (\bar{\Psi}^C)^{\alpha}_i \gamma_5 \Psi^\beta_j \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta \]

- Symmetry of ground state (2SC phase):
  - chiral \(\text{SU}(2)_L \times \text{SU}(2)_R\) — intact
  - baryon number \(\text{U}(1)_B \rightarrow \tilde{\text{U}}(1)_B\) with \(\tilde{B} = B - \frac{2}{\sqrt{3}} T_8\)
  - approximate axial \(\text{U}(1)_A\) is broken \(\rightarrow\) 1 pseudo-NG boson
  - Color gauge symmetry \(\text{SU}(3)_c \rightarrow \text{SU}(2)_c\) by Anderson-Higgs mechanism \(\rightarrow\) 5 massive gluons
  - Gauge symmetry \(\text{U}(1)_{em} \rightarrow \tilde{\text{U}}(1)_{em}\) with \(\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8\)
Physical properties of 2SC phase

- Pressure/equation of state:
  \[ P \simeq \frac{\mu^4}{2\pi^2} - B + \frac{\mu^2 \Delta^2}{\pi^2} \]
  may be (un-)important

- Transport/specific heat is dominated by
  - Unpaired “blue-up” and “blue-down” quarks
  - 1 pseudo-NG boson that results from breaking \( U(1)_A \)
  - 3 gluons of unbroken \( SU(2)_c \) (decoupled from blue quarks)
  - low energy photon of \( \tilde{U}(1)_{em} \)

- No superfluidity → no rotational vorticies

- No electromagnetic Meissner effect → no magnetic flux tubes

- Neutrino emissivity/cooling rate is large (direct URCA)
Color superconductivity, $N_f = 3$

- Wave function of a Cooper-pair:
  Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\mathbb{Z}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_{\mathbb{Z}}$

- Diquark condensate (spin-0 gap $\sim 10 - 100$ MeV):
  $$\langle (\bar{\Psi}_L^C)^\alpha_i (\Psi_L^C)^\beta_j \rangle = \langle (\bar{\Psi}_R^C)^\alpha_i (\Psi_R)^\beta_j \rangle \simeq \sum \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- Color-flavor locking: $\text{SU}(3)_L \times \text{SU}(3)_R \times \text{SU}(3)_c \rightarrow \text{SU}(3)_{L+R+c}$
  [Alford et al. hep-ph/9804403]

There are no $\langle q_L q_R \rangle$ condensates, but $\text{SU}(3)_L \times \text{SU}(3)_R$ chiral symmetry is broken down to $\text{SU}(3)_V$ through locking with color!

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Color superconductivity, $N_f = 3$

- Symmetry of ground state (CFL phase):
  - chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
    $\rightarrow$ 8 (pseudo-)NG bosons, i.e., $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$
  - baryon number $U(1)_B$ is broken $\rightarrow$ 1 NG boson ($\phi$)
  - approximate axial $U(1)_A$ is broken $\rightarrow$ 1 pseudo-NG boson ($\eta'$)
  - Color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism $\rightarrow$ 8 massive gluons
  - Gauge symmetry $U(1)_{em} \rightarrow \tilde{U}(1)_{em}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$

- Quasiparticle spectrum (9 quark quasiparticles):
  - octet under $SU(3)_{L+R+c}$ with gap $\Delta$
  - singlet under $SU(3)_{L+R+c}$ with gap $2\Delta$
Physical properties of CFL phase

- Pressure/equation of state:
  \[ P \simeq \frac{3\mu^4}{4\pi^2} - B + \frac{3\mu^2\Delta^2}{\pi^2} \]
  may be (un-)important

- Transport/specific heat is dominated by [Shovkovy & Ellis, 2002]
  - 1 NG boson (\(\phi\)) that results from breaking \(U(1)_B\)
  - low energy photon of \(\tilde{U}(1)_{em}\)
  - 1 pseudo-NG boson (\(\eta'\)) that results from breaking \(U(1)_A\)
  - 8 (\(\times 3\) polarizations) light plasmons with mass \(\sim \Delta\) (?)
    [Gusynin & Shovkovy, hep-ph/0108175]

- Superfluidity \(\rightarrow\) rotational vorticities

- No electromagnetic Meissner effect \(\rightarrow\) no magnetic flux tubes

- Neutrino emissivity/cooling rate is suppressed (\(\sim e^{-\Delta/T}\))
Color superconductivity, $N_f = 1$

- Wave function of a Cooper-pair: $\langle \bullet \bullet \rangle - \langle \bullet \bullet \rangle \otimes \uparrow \uparrow \rangle_{J=1}$
  - antisymmetric in color (attractive diquark $3_c$ channel)
  - Pauli principle: symmetric in spin, i.e., spin-1 channel
- Diquark condensate (gap $\sim 10 - 100$ keV):
  \[
  \langle (\bar{\Psi}^C)^{\alpha} \gamma_5 \Psi^{\beta} \rangle \simeq \varepsilon^{\alpha \beta \gamma} \Delta_{c \delta} \left( \hat{k}^\delta \sin \theta + \gamma^\delta_\perp (\hat{k}) \cos \theta \right)
  \]
- Many possibilities, see Ph.D. thesis [A.Schmitt, nucl-th/0405076]:
  - Polar phase: $\Delta_{c \delta} = \delta_{c3} \delta_{33}$
  - Color-spin-locked phase: $\Delta_{c \delta} = \delta_{c \delta}$
  - Planar phase: $\Delta_{c \delta} = \delta_{c \delta} - \delta_{c3} \delta_{33}$
  - A-phase: $\Delta_{c \delta} = \delta_{c3} (\delta_{\delta1} + i \delta_{\delta2}) \rightarrow$ largest pressure
- Meissner effect $\oplus$ type I superconductor $\rightarrow$ affect star properties
Color superconductivity in stars

Is there CSC matter inside compact stars?

Arguments in favor:

 questões Relatively high densities in stars, $\rho_c \gtrsim 5\rho$, suggest that quarks may be deconfined
 questões An attractive diquark channel is likely to exist
 questões Temperatures are quite low, $T \lesssim 50$ keV, to allow pairing

Arguments against:

 questões Strongly coupled dynamics is not under control
 questões Matter may not necessarily be deconfined at existing densities
 questões Specific conditions inside stars (e.g., $\beta$-equilibrium) may not favor color superconductivity

The natural approach: to give model predictions and to test them

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Importance of neutrality inside a star

Matter in the bulk of a star is

(i) $\beta$-equilibrated: $\mu_d = \mu_u + \mu_e$
(ii) electrically and color neutral:
$$n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$$

Otherwise, a star is not stable!

- Coulomb energy (when $n_Q \neq 0$)

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left( \frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left( \frac{R}{1 \text{ km}} \right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2\text{SC}} \gg M_\odot c^2$
Neutrality vs. color superconductivity

- The “best” 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in $\beta$-equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \implies \mu_d \approx 2^{1/3}\mu_u \implies \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

i.e.,

$$n_e \approx \frac{1}{4^3\frac{n_u}{3}} \ll n_u$$

The “best” Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$
Appearance of a gapless phase

Mismatch parameter $\mu_e$ is not a free model parameter,

$$n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0 \quad \Rightarrow \quad \mu_e = \mu_e(\mu, \Delta)$$

Three dynamical regimes determined by coupling strength $\eta$:

1. $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing:
   
   normal phase is the ground state

2. $\eta \gtrsim 0.8$ — strong coupling wins over the mismatch between the Fermi surfaces: 2SC is the ground state

3. $0.7 \lesssim \eta \lesssim 0.8$ — regime of intermediate coupling strength:
   
   the ground state is the gapless 2SC phase [hep-ph/0302142]
Quasiparticle spectrum in normal phase

“Weak” coupling (normal phase)

How does this spectrum change when Cooper pairs are formed?
Quasiparticle spectrum in 2SC phase

“Strong” coupling (2SC phase)

The energy gaps in the quasiparticle spectra are $\Delta - \delta \mu$ & $\Delta + \delta \mu$
Quasiparticle spectrum in g2SC phase

“Intermediate” coupling (gapless phase)

The energy gaps in the quasiparticle spectra are $0$ & $\Delta + \delta \mu$
Stability of g2SC phase


$$V(\text{MeV/fm}^3)$$

-81
-82
-83
-84
-85
-86
-87

$n_Q=0$

$\mu_e=148 \text{ MeV}$

$\Delta(\text{MeV})$

20 40 60 80 100 120

$\sigma \gtrsim 20 \text{ MeV/fm}^2$ [Shovkovy, Hanuske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054]

No Sarma instability if $n_Q = 0$ is enforced locally!

Recent results for gluon screening masses
[Huang & Shovkovy, hep-ph/0407049]:

\[ m_b^2 / m_g^2 \]

\[ \Delta / \delta \mu \]

\[ m_{\tilde{b}}^2 / m_g^2 \]

\[ \Delta / \delta \mu \]

\[ m_{\tilde{b}}^2 < 0 \]

\[ A = 1, 2, 3 \quad \text{— red solid line} \]

\[ A = 4, 5, 6, 7 \quad \text{— green long-dash line} \]

\[ A = \tilde{8} \quad \text{— blue short-dash line} \]
Finite strange quark mass, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \sim \mu - \frac{m_s^2}{2\mu}$$

The ground state of strange quark matter may have:

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystalline pairing (nonzero momentum pairing, LOFF)

Recently, other possibilities were proposed as well ...
**Gapless $N_f = 3$ quark matter**

- Distorted color-flavor pairing:
  
  $$
  \Delta_{ij}^{\alpha\beta} \simeq \Delta_1 \epsilon_{1ij} \epsilon^{1\alpha\beta} + \Delta_2 \epsilon_{2ij} \epsilon^{2\alpha\beta} + \Delta_3 \epsilon_{3ij} \epsilon^{3\alpha\beta} + \ldots
  $$

- Control (mismatch) parameter:
  
  $$
  \delta \mu \equiv \frac{\mu_{bd} - \mu_{gs}^{\text{eff}}}{2} \approx -\frac{\mu_8}{2} + \frac{m_s^2}{4\mu} \approx \frac{m_s^2}{2\mu}
  $$

  where $\mu_{gs}^{\text{eff}} \simeq \mu_{gs} - \frac{m_s^2}{2\mu}$ and $\mu_8 \simeq -\frac{m_s^2}{2\mu}$ (blue color is special)

- Gapless CFL phase with $\Delta_1 < \Delta_2 < \Delta_3$:
  
  $$
  T = 0 : \quad \delta \mu \equiv \frac{m_s^2}{2\mu} > \Delta_0 \quad [\text{Alford et al. hep-ph/0311286}]
  $$
Quasiparticle spectrum in gCFL phase

a) $|k_0| [\text{MeV}]$ vs $k [\text{MeV}]$ with lines $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$, and $\tilde{\varepsilon}_3$.

b) $|k_0| [\text{MeV}]$ vs $k [\text{MeV}]$ with lines $k_{0d}^{rg}$ and $k_{0d}^{gr}$.

c) $|k_0| [\text{MeV}]$ vs $k [\text{MeV}]$ with lines $k_{0s}^{rb}$ and $k_{0s}^{br}$.

d) $|k_0| [\text{MeV}]$ vs $k [\text{MeV}]$ with lines $k_{0s}^{gb}$ and $k_{0s}^{bg}$.
Nonzero temperature

- There can exist many phases at $T \neq 0$
  

- Zoo of phases:

  - **CFL:** $\Delta_1 \neq 0$, $\Delta_2 \neq 0$, $\Delta_3 \neq 0$, ($\mu_e \approx 0$)
  - **mCFL:** $\Delta_1 \neq 0$, $\Delta_2 \neq 0$, $\Delta_3 \neq 0$, ($\mu_e \neq 0$)
  - **uSC:** $\Delta_1 = 0$, $\Delta_2 \neq 0$, $\Delta_3 \neq 0$,
  - **dSC:** $\Delta_1 \neq 0$, $\Delta_2 = 0$, $\Delta_3 \neq 0$,
  - **2SC:** $\Delta_1 = 0$, $\Delta_2 = 0$, $\Delta_3 \neq 0$,
  - **NQM:** $\Delta_1 = 0$, $\Delta_2 = 0$, $\Delta_3 = 0$,

  plus g2SC and gCFL as special cases of 2SC and mCFL.
Overview of phases with strangeness

• Color-flavor locked (CFL) phase
  – “Enforced pairing”: $n_u = n_d = n_s \ (T \sim 0)$
    [Rajagopal, Wilczek, 2001]
  – Natural insulator, $n_{e_1} \sim 0$
  – Little specific heat and low neutrino emissivity

• Metallic CFL phase ($n_{e_1} \neq 0$)
  – $T = 0$: gapless CFL phase (no “enforced pairing”)
  – $T \neq 0$: thermal effects $\rightarrow n_{e_1} \neq 0$
  – Large specific heat and high neutrino emissivity

• uSC phase: only $ud$- & $us$-pairing (no $ds$-pairing)

• dSC phase: only $du$- & $ds$-pairing (no $us$-pairing)
Phase diagram

a) \( \mu = 500 \text{ MeV} \)

b) \( m_s = 250 \text{ MeV} \)
Current status

- Sufficiently cold and dense matter is a color superconductor

- Neutrality and $\beta$-equilibrium may strongly affect the properties of dense matter

- There can exist many different CSC phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+$K^0$, CFL+$\eta$)

- Some features of $T - \mu$ phase diagram start to develop

- A search for signature-type observables of color superconductivity inside stars is under way
Outlook

• A systematic study of competition between different phases in dense QCD should be completed

• Physical properties (transport, in particular) of QCD phases should be addressed in detail

• The status of gapless phases should be resolved (addressing, e.g., the chromomagnetic instability, spontaneously induced currents)

• The most promising observable(s), (dis-)proving the presence of CSC inside stars, should be proposed

• A rigorous approach to treat QCD at nonzero densities should be developed