Stable gapless color superconducting phases
of dense quark matter

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References

Matter at high density

Why do we study this?
(i) fundamental properties of QCD
(ii) properties of compact stars

(i) No lattice results, $\mu_q \gg \Lambda_{QCD}$
(ii) Densities in stars $\rho_c \gtrsim 5\rho_0$
Color superconductivity

Dense quark matter is a color superconductor. [Barrois,78], [Bailin&Love,84]

Simplest case, 2SC

- $N_f = 2$: “up” and “down” quarks
- $p_F^{up} = p_F^{down} = \mu_q$
- $T = 0$: ground state – Fermi gas ($\alpha_s = 0$)

\[
\left\langle u_p \, d_{-p}\right\rangle = - \left\langle u_q \, d_{-q}\right\rangle \neq 0
\]

\[
\begin{array}{c}
\vec{p}_F \\
\vec{q}_F
\end{array} \quad \begin{array}{c}
u \\
d
\end{array}
\]

Cooper instability $\rightarrow$ color superconductivity

\[
\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)_3 \otimes \left(\left|\uparrow\uparrow\right\rangle - \left|\downarrow\downarrow\right\rangle\right)_0 \otimes (|u,d\rangle - |d,u\rangle)
\]
Matter in the bulk of a star should be

(i) electrically and color neutral:

\[ Q_{\text{el}} = 0, \quad Q_{\text{color}} = 0 \]

(ii) \( \beta \)-equilibrated: \( \mu_d = \mu_u + \mu_e \)

\[ \downarrow \]

\[ p_F^{\text{up}} \neq p_F^{\text{down}} \]

\[ \downarrow \]

The “best” Cooper pairing is distorted:

The mismatch parameter is

\[ \delta \mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0 \]
**Regular 2SC vs. gapless 2SC**

“Small” mismatch ($\delta \mu < \Delta$)  
“Large” mismatch ($\delta \mu > \Delta$)

Energy gaps:  
$$(\Delta - \delta \mu) \& (\Delta + \delta \mu)$$

Energy gaps:  
$$0 \& (\Delta + \delta \mu)$$
Stability of g2SC phase

Eff. potential at $T = 0$

Eff. potential at $T \neq 0$

(Mixed phase? — unlikely)

(Note: 2nd order phase transition)

Thus, g2SC is stable provided $Q = 0$ is enforced locally

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• **Nonmonotonic** temperature dependence

• **Transitional behavior**: $g2SC \rightarrow 2SC \rightarrow g2SC \rightarrow \text{normal phase}
Temperature dependence of the gap. II.

- Extreme nonmonotonic temperature dependence
- Transitional behavior: normal phase $\rightarrow$ g2SC $\rightarrow$ normal phase
• The ratio is not universal (unlike in BCS)

• The value of $T_c/\Delta_0$ can be arbitrarily large
Summary

- The g2SC phase is a new state of matter that may exist in cores of compact stars
- The g2SC phase is stable if the neutrality is enforced locally
- The spectrum of low-energy excitations in the g2SC phase has extra gapless modes (these should affect transport properties)
- Temperature dependence of the gap is nonmonotonic
- Ratio $T_c/\Delta_0$ is nonuniversal, and can be arbitrarily large
- A gapless phase of $N_f = 3$ quark matter is also possible
- Similar gapless phases may appear in asymmetric nuclear matter and in trapped cold gases of fermionic atoms (e.g., $^6$Li and $^{40}$K)
[1] Unstable Sarma phase:

[2] Asymmetric nuclear matter:

[3] Interior gap superfluidity (atomic gases):

[4] Breached pairing superfluidity:

[5] Gapless color flavor locked phase: