Gapless color superconductivity

Igor A. Shovkovy

Institut für Theoretische Physik
J. W. Goethe Universität
Frankfurt am Main
Collaborator(s)

- Mei Huang
  
  (ITP, Goethe-University & Tsinghua University, Beijing)

References


Matter at high density

Why do we study this?
(i) fundamental properties of QCD
(ii) properties of compact stars

(i) No lattice results, $\mu_q \gg \Lambda_{QCD}$
(ii) Densities in stars $\rho_c \gtrsim 5\rho_0$
Dense quark matter is a color superconductor. [Barrios, 78], [Bailin & Love, 84]

**Simplest case, 2SC**

- $N_f = 2$: “up” and “down” quarks
- $p_F^{\text{up}} = p_F^{\text{down}} = \mu_q$
- $T = 0$: ground state – Fermi gas ($\alpha_s = 0$)

\[
\frac{3}{2} a + 6_s
\]

Cooper instability → color superconductivity

\[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

\[
\left( |\uparrow\rangle - |\downarrow\rangle \right) \otimes \left( |\uparrow\rangle - |\downarrow\rangle \right) \otimes \left( |u,d\rangle - |d,u\rangle \right)
\]

\[
\left( |\uparrow\rangle - |\downarrow\rangle \right) \otimes \left( |\uparrow\rangle - |\downarrow\rangle \right) \otimes \left( |u,d\rangle - |d,u\rangle \right)
\]
Neutralty vs. color superconductivity

Matter in the bulk of a star should be

(i) electrically and color neutral:
\[ Q_{\text{el}} = 0, \quad Q_{\text{color}} = 0 \]

(ii) $\beta$-equilibrated: $\mu_d = \mu_u + \mu_e$

\[ \downarrow \]

\[ p_F^{\text{up}} \neq p_F^{\text{down}} \]

\[ \downarrow \]

The “best” Cooper pairing is distorted:

The mismatch parameter is
\[ \delta \mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0 \]
Regular 2SC vs. gapless 2SC

“Small” mismatch ($\delta \mu < \Delta$)  “Large” mismatch ($\delta \mu > \Delta$)

Energy gaps:
($\Delta - \delta \mu$) & ($\Delta + \delta \mu$)  0 & ($\Delta + \delta \mu$)
Stability of g2SC phase

Eff. potential at $T = 0$

Eff. potential at $T \neq 0$

(Mixed phase? — unlikely)  
(Note: 2nd order phase transition)

Thus, g2SC is stable provided $Q = 0$ is enforced *locally*
Temperature dependence of the gap. I.

- Nonmonotonic temperature dependence
- Transitional behavior: g2SC → 2SC → g2SC → normal phase
• Extreme nonmonotonic temperature dependence

• Transitional behavior: normal phase $\rightarrow$ g2SC $\rightarrow$ normal phase
The ratio is not universal (unlike in BCS)

The value of $T_c/\Delta_0$ can be arbitrarily large
Summary

- The g2SC phase is a new state of matter that may exist in cores of compact stars.
- The g2SC phase is stable if the neutrality is enforced locally.
- The spectrum of low-energy excitations in the g2SC phase has extra gapless modes (these should affect transport properties).
- Temperature dependence of the gap is nonmonotonic.
- Ratio $T_c/\Delta_0$ is nonuniversal, and can be arbitrarily large.
- A gapless phase of $N_f = 3$ quark matter is also possible.
- Similar gapless phases may appear in asymmetric nuclear matter and in trapped cold gases of fermionic atoms (e.g., $^6$Li and $^{40}$K).
Additional references

[1] Unstable Sarma phase:

[2] Asymmetric nuclear matter:

[3] Interior gap superfluidity (atomic gases):

[4] Breached pairing superfluidity:

[5] Gapless color flavor locked phase: