Color superconductivity and compact stars

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References

Weakly interacting quark matter

The property of asymptotic freedom: \( \alpha_s(\mu) \ll 1 \) for \( \mu \gg \Lambda_{QCD} \)

[Gross & Wilczek, 73], [Politzer, 73]

- Dense quark matter is weakly interacting [Collins & Perry, 75]
- "Squeezing" quark matter

- Realistic densities in compact stars: \( \rho \approx 10\rho_0 \), where \( \rho_0 \approx 0.15 \text{ fm}^{-3} \),
  (corresponding coupling \( \alpha_s \sim 1 \))

\[\begin{array}{|c|c|c|}
\hline
\text{Data} & \text{Theory} & \text{NLO} \\
\hline
\text{Deep Inelastic Scattering} & NLO & \text{Lattice} \\
\text{e+e Annihilation} & & \text{Lattice} \\
\text{Hadron Collisions} & & \text{Lattice} \\
\text{Heavy Quarkonia} & & \text{Lattice} \\
\hline
\end{array}\]

\[
\begin{align*}
\alpha_s(M^2) & \approx 0.1210 \\
\alpha_s(M^2) & \approx 0.1156 \\
\alpha_s(M^2) & \approx 0.1183 \\
\alpha_s(M^2) & \approx 0.1156
\end{align*}
\]

\[Q \approx 10 \rho_0, \quad \rho_0 \approx 0.15 \text{ fm}^{-3},
\] (corresponding coupling \( \alpha_s \sim 1 \))

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Basics of color superconductivity

Asymptotic density ($\mu \gg \Lambda_{QCD}$):

- $\alpha_s(\mu) \ll 1$ (weak coupling)
- One-gluon interaction is dominant
- Color $\bar{3}_a$ channel is attractive (!)

- BCS mechanism for quarks leads to color superconductivity
- By using Pauli principle ($s = 0$):

$$N_f = 2 : \varepsilon_{ij}\varepsilon^{ab}_{ij} \langle (\psi^i_a)^T C\gamma^5 \psi^j_b \rangle \neq 0$$

$$N_f = 3 : \sum_{I=1}^{3} \varepsilon_{ijI}\varepsilon^{abI}_{ij} \langle (\psi^i_a)^T C\gamma^5 \psi^j_b \rangle \neq 0$$

[Son], [Schafer et al.], [Hong et al.], [Pisarski et al.], [Shovkovy et al.] (1999)
Properties of 2SC ground state
(up & down quarks only)

- Chiral symmetry $SU(2)_L \times SU(2)_R$ is intact
- Color symmetry is broken (by Anderson-Higgs mechanism):
  $SU(3)_c \rightarrow SU(2)_c$
  - color Meissner effect (for 5 gluons)
  - low energy $SU(2)_c$ gluodynamics (decoupled)
- Modified electromagnetic $U(1)_{\text{em}}$ and modified $U(1)_{\overline{B}}$ survive
  - no electromagnetic Meissner effect
  - no superfluidity
- Approximate $U(1)_A$ is broken $\rightarrow$ light pseudo-NG boson
- Parity is preserved
Signatures of CSC in compact stars

Color superconductivity $\rightarrow$ **gap** in quasiparticle spectrum

- Thermodynamic properties (equation of state)
  - mass-radius relation [Alford&Reddy,02], [Lugones&Horvath,02]
  - internal star structure [Baldo et al.,02], [Shovkovy et al.,03]

- Transport properties (conductivities, viscosities, mean free paths)
  - cooling rate [Page et al.,02], [Shovkovy&Ellis,02]
  - r-mode instability [Madsen,99]
  - glitches (crystalline phase) [Alford et al.,00]

- Other properties
  - magnetic field generation/penetration [Alford et al.,00]
  - rotational vortices [Iida&Baym,02]
Neutrality vs. color superconductivity

- The “best” 2SC phase appears when $n_d \approx n_u$,
- but neutral matter appears when $n_d \approx 2n_u$
- Electrons do not help (!):

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3}\mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u$$

Thus, $n_e \approx \frac{1}{4^3\frac{1}{3}} \ll n_u$

- Cooper pairing with a mismatch between Fermi surfaces of pairing quarks:

$$\mu_d - \mu_u = \mu_e$$

Gaps: $(\Delta + \mu_e/2)$ and $(\Delta - \mu_e/2)$
2SC vs. gapless 2SC phase

- Either $\delta \mu < \Delta$, or $\delta \mu > \Delta$
- Gapless 2SC is a *stable* phase of neutral matter in $\beta$-equilibrium

[I.A.S. & M. Huang, hep-ph/0302142]

- Extra 2 gapless quasiparticles
Neutral quark phases

- Locally neutral phases:
  - Normal quark matter
  - gapless 2SC matter [Shovkovy&Huang,03]

- Globally neutral mixed phases [Glendenning,92], e.g., 2SC+NQ (?)

\[ \rho_e^{(MP)} = \chi_B^A \rho_e^{(A)}(\mu, \mu_e) + (1 - \chi_B^A) \rho_e^{(B)}(\mu, \mu_e) = 0 \]

where

\[ \chi_B^A \equiv \frac{V^{(A)}}{V^{(A)} + V^{(B)}} \] is the volume fraction of phase A
Gibbs construction \((2\text{SC}+\text{NQ})\)

- Mechanical equilibrium:
  \[ P^{(2\text{SC})}(\mu, \mu_e) = P^{(\text{NQ})}(\mu, \mu_e) \]

- Chemical equilibrium:
  \[ \mu = \mu^{(2\text{SC})} = \mu^{(\text{NQ})}, \]
  \[ \mu_e = \mu_e^{(2\text{SC})} = \mu_e^{(\text{NQ})} \]

- From the condition of neutrality
  \[ \chi^{2\text{SC}}_{\text{NQ}} = \frac{\rho_e^{(\text{NQ})}}{\rho_e^{(\text{NQ})} - \rho_e^{(2\text{SC})}}, \]

Energy density: \(\varepsilon^{(\text{MP})} = \chi^{2\text{SC}}_{\text{NQ}} \varepsilon^{(2\text{SC})}(\mu, \mu_e) + (1 - \chi^{2\text{SC}}_{\text{NQ}})\varepsilon^{(\text{NQ})}(\mu, \mu_e)\)

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Coulomb forces and surface tension

- Extra surface and Coulomb energies per unit volume
  [Heiselberg et al.,93], [Glendenning & Pei,95]

\[ \mathcal{E}_S \simeq C_S^{(\text{geom})} \frac{\sigma}{R}, \quad \mathcal{E}_C \simeq C_C^{(\text{geom})} \left( \rho_e^{(A)} - \rho_e^{(B)} \right)^2 R^2 \]

- Minimizing the sum with respect to \( R \), one gets

\[ \mathcal{E}_{C+S} \simeq (8 \text{ MeV } \text{fm}^{-3}) \left( \frac{\sigma}{\sigma_0} \frac{\rho_e^{(A)} - \rho_e^{(B)}}{\rho_e^{(0)}} \right)^{2/3}, \quad \text{ (“slabs”)} \]

where \( \sigma_0 = 50 \text{ MeV } \text{fm}^{-2} \) and \( \rho_e^{(0)} = 0.4e \text{ fm}^{-3} \)

- Thickness of “slabs”

\[ a \simeq (9.4 \text{ fm}) \left( \frac{\sigma}{\sigma_0} \right)^{1/3} \left( \frac{\rho_e^{(0)}}{\rho_e^{(A)} - \rho_e^{(B)}} \right)^{2/3}, \]
Coulomb and surface effects in 2SC+NQ matter

Coulomb effects are easy to estimate, while surface tension is usually not well known.

There are three possible cases:

- **Low surface tension** ($\sigma \lesssim 20$ MeV fm$^{-2}$):
  little effect; mixed phase survives ("slabs" with $a \simeq 10$ fm)

- **Intermediate values of surface tension** ($20 \lesssim \sigma \lesssim 50$ MeV fm$^{-2}$):
  phase transition occurs at higher densities, $3\rho_0 \lesssim \rho_B \lesssim 5\rho_0$

- **Large values of surface tension** ($\sigma \gtrsim 50$ MeV fm$^{-2}$):
  homogeneous phase is more favorable than mixed phase

Similar estimates are valid for hadron-quark mixed phases
[Heiselberg et al.,01], [Alford et al.,01]

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**Hadronic matter**

- At low densities $\sim \rho_0$ quarks are confined
- Some hadronic description is required
- We use hadronic chiral $SU(3)_L \times SU(3)_R$ model
  - [Papazoglou, 98], [Papazoglou, 99], [Hanauske, 00]
  - nonlinear realization of $SU(3)_L \times SU(3)_R$
  - spontaneous symmetry breaking
  - small explicite symmetry breaking
  - QCD motivated dilaton field is included
- Model describes well hadronic masses, finite nuclei, hypernuclei and neutron star properties
Hybrid matter

Hadronic phase $\rightarrow$ Hadron-quark MP $\rightarrow$ 2SC+NQ quark MP

- Star crust matter:
  $\rho_B \leq 0.08 \text{ fm}^{-3}$ [Baym et al., 71], [Negele & Vautherin, 73]

- Hadronic matter:
  $0.08 \leq \rho_B \leq 1.49 \text{ fm}^{-3}$

- Hadron-quark mixed phase:
  $1.49 \leq \rho_B \leq 2.56 \text{ fm}^{-3}$

- 2SC+NQ quark mixed phase:
  $\rho_B \geq 2.75 \text{ fm}^{-3}$

$\triangle$-point is a triple point (!)
Equation of state

Star structure is determined by metric that satisfies Tolman-Oppenheimer-Volkoff equations

**Input:** equation of state $P(\varepsilon)$

- $\Box$ – beginning of hadron-quark MP
- $\Delta$ – beginning of 2SC+NQ quark MP

- $\varepsilon$ and $\rho_B$ have jumps at the triple point
Compact star structure

- $\epsilon_c = 210 \text{ MeV/fm}^3$ – pure hadronic star;
- $\epsilon_c = 370 \text{ MeV/fm}^3$ – hybrid star without quark core;
- $\epsilon_c = 500 \text{ MeV/fm}^3$ – hybrid star with a quark core;
- $\epsilon_c = 1392 \text{ MeV/fm}^3$ – largest mass star with parameters:
  - $M_{\text{max}} = 1.81 M_\odot$
  - $\rho_c = 7.58 \rho_0$
  - $R = 10.86 \text{ km}$

There are no stars with $378 \leq \epsilon_c \leq 415 \text{ MeV/fm}^3$
**Mass-radius relation**

Stars heavier than “◯-stars” may have strange matter in their cores.
Summary

- Realistic EoS of nonstrange hybrid baryon matter is constructed
- Charge neutrality and $\beta$-equilibrium are taken into account; they play very important role
- Two-flavor color superconducting matter appears naturally as a component of 2SC+NQ mixed phase
- Construction of 2SC+NQ mixed phase is very stable: volume fractions of components change little with changing density
- The first example of a triple point is obtained and studied
- Sharp interface between the two mixed phases is observed; this is smoothed over distances of about 10 fm
Outlook

- Generalization including strange quarks is needed
- All kinds of hybrid star properties should be studied
  - neutrino emissivity and mean free path
  - cooling rates of mixed phases
  - magnetic properties
- Surface tension effects and screening of Coulomb forces should be addressed
- Studies of color superconductivity in rotating stars are of interest
- Search for signatures of color superconductivity in compact stars should be made systematic