Microscopic approach to color superconductivity of dense quark matter

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References:

- hep-ph/0104194
- hep-ph/0103269
Outline

1. Introduction

2. Weakly interacting dense quark matter

3. Gap equation, gluon proparagor, etc.

4. S2C: pseudo-NG bosons

5. Mass estimates of pNGBs

6. CFL: Collective excitations

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**Introduction**

- **Phase diagram of QCD**

  “It [Holy Grail] promises mystery, secrecy, adventure and the obtaining of a prize or knowledge available to all but found by few”

![Phase diagram of QCD](image)

- **Interior of neutron stars** (neutrino emission, cooling rate, magnetic field, thermodynamics, R-mode instability, glitches, γ-ray bursts, etc.)

- **Heavy ion collisions (?)**
Weakly interacting dense quark matter

- Densities $\gtrsim 3n_0$, where $n_0 \approx 0.17\text{fm}^{-3}$
  
  - "Squeezing" quark matter

- Asymptotic freedom
  [Gross, Wilczek, Politzer, '73]

- Weakly interacting quarks
  [Collins and Perry, '75]
• Coupling constant: $\alpha(\mu) \ll 1$
  (where $\mu \gg \Lambda_{QCD}$)

• One-gluon exchange is the dominant interaction between quarks [Bailin & Love,'84]

\[
\begin{array}{c}
\text{p} \\
\text{k} \\
\text{p-k} \\
\text{-p} \\
\text{-k}
\end{array}
\begin{array}{c}
\Rightarrow \\
3 + 6
\end{array}

• Antisymmetric $\bar{3}$ channel is attractive (!)

• Cooper instability around the Fermi surface

• Diquark condensate

\[
\begin{align*}
N_f = 2 : & \quad \epsilon_{ij} \epsilon^{ab}_3 \langle (\psi_i^a)^T C \gamma^5 \psi_b^j \rangle \neq 0 \\
N_f = 3 : & \quad \sum_{I=1}^{3} \epsilon_{ij} \epsilon^{aIb} \langle (\psi_i^a)^T C \gamma^5 \psi_b^j \rangle \neq 0
\end{align*}
\]

• Symmetries (note, parity is preserved)

\[
\begin{align*}
N_f = 2 : & \quad SU(2)_L \times SU(2)_R \times SU(2)_c \\
N_f = 3 : & \quad SU(3)_{L+R+c}
\end{align*}
\]
The value of the gap

- **If** the gluon interaction were *screened* at the scale of $m_D \sim \sqrt{\alpha \mu}$ [Bailin & Love,’84], then
  \[
  \Delta \sim \mu \exp \left( -\frac{C}{\alpha} \right) \rightarrow \Delta \ll 1 \text{ MeV}
  \]

- Phenomenological 4-fermion models [Alford,Rajagopal&Wilczek; Rapp,Schäfer,Shuryak&Velkovsky,’88] give
  \[
  \Delta \sim 50 \text{ to } 100 \text{ MeV}
  \]

- Proper perturbative QCD analysis:
  - There is only *dynamical screening* for magnetic gluons [Son,’99]
  - Method of Schwinger-Dyson equation
    [Hong,Miransky,Shovkovy&Wijewardhana,’00]:

\[
\Delta \sim \frac{(4\pi)^{3/2} \mu}{\sqrt{\alpha(\mu)}} \frac{C}{\alpha^{5/2}(\mu)} \sim 20 \text{ to } 70 \text{ MeV}
\]
Gluon propagator (HDL)

- Region of dominant interaction:
  \[ |\Delta| \ll |k_4| \ll |\vec{k}| \ll \mu \]

- Dynamical screening (magnetic modes)

\[
iD_{\mu\nu} \simeq \frac{|\vec{k}|O_{\mu\nu}^{(mag)}}{|\vec{k}|^3 + \pi M_D^2 |k_4|/2} - \frac{O_{\mu\nu}^{(el)}}{k_4^2 + |\vec{k}|^2 + 2M_D^2},
\]

where \( M_D^2 = \alpha_s N_f \mu / \pi \), and

\[
O_{\mu\nu}^{(mag)}(q) = g_{\mu\nu} - u_\mu u_\nu + \frac{\vec{q}_\mu \vec{q}_\nu}{|q|^2},
\]

\[
O_{\mu\nu}^{(el)}(q) = u_\mu u_\nu - \frac{\vec{q}_\mu \vec{q}_\nu}{|q|^2} - \frac{q_\mu q_\nu}{q^2},
\]

where \( u_\mu = (1,0,0,0) \) and \( \vec{q}_\mu = q_\mu - (u \cdot q) u_\mu \).

- Meissner effect \((|k_4|, |\vec{k}| \lesssim |\Delta|)\) is irrelevant

- No strong coupling effects for \( \mu \gg \Lambda_{QCD} \)

\[
\Lambda_{QCD} \ll |\Delta| \sim (\ln \mu)^{-5/2} e^{\ln \mu - C \sqrt{b \ln \mu}}
\]

- Note: no decoupling of strong IR dynamics in lower dimensions \((d=2+1 \text{ or } d=1+1)\)
**S2C: Pseudo-NG bosons**

- **Approximate** symmetries of dense QCD:
  - $U(1)_A$ [in addition to the exact $U(1)_V$]
  - $SU(3)_{cL} \times SU(3)_{cR}$ [replaces $SU(3)_c$]

- **Explanation**:
  - Anomaly is negligible at large $\mu$ ($\gg \Lambda_{QCD}$)
  - Leading order kernel of BS equation does not mix L- and R-handed quarks

- **Approximate** symmetry breaking partern:
  - $SU(3)_{cL} \times SU(3)_{cR} \rightarrow SU(2)_{cL} \times SU(2)_{cR}$
  - $U(1)_A$ is broken [mixed with axial color]
  - $\tilde{U}(1)_{V}$ is intact [generator $\sim \frac{1}{3}I - \frac{2}{\sqrt{3}}T^8$]

- **Total number of pNGBs?**
  - $5$ (scalars) $\oplus 5$ (pseudoscalars)
  [taking Higgs mechanism into account]
Quantum numbers & properties of pNGBs

- $U(1)_A$ generator: $\gamma^5 I$ [or $\frac{2}{3}I + \frac{2}{\sqrt{3}}T^8$]
  1 pseudoscalar: SU(2)$_c$ singlet

- Generators: $\gamma^5 T^A$, $A = 4, 5, 6, 7$
  4 pseudoscalars: SU(2)$_c$ doublet and antiduallet [generators $\sim \gamma^5 (T^4 \pm T^5)$ & $\gamma^5 (T^6 \pm T^7)$]

- Bethe-Salpeter equation:

- Decay constants and velocities:

$$F = \frac{\mu}{2\sqrt{2\pi}}, \quad v = \frac{1}{\sqrt{3}}$$
Masses of the pNGBs

- Add axial color phases to the gap function
  \[ \Delta \to \mathcal{P}_+ U^\dagger \Delta U^* + \mathcal{P}_- U \Delta U^T, \]
  where \( U = \exp(i \gamma^5 \omega^{TA}) \)

- Effective potential becomes a function of the \( U \) field
  \[ V(\Delta) \to V(\Delta, U) \]

- Masses of the pNGBs:
  \[ M_x^2 \sim \frac{1}{F^2} \left. \frac{\partial^2 V(\Delta, U)}{\partial \omega_x^2} \right|_{\omega^A = 0} \]

- Vacuum energy diagrams:
  
  (a) \hspace{1cm} + \hspace{1cm} (b) \hspace{1cm} + \hspace{1cm} (c)

  \hspace{1cm}

  (d) \hspace{1cm} + \hspace{1cm} (e)
Results

• Quadratic term of the vacuum energy:

\[ \delta \omega E_{\text{vac}} \simeq \frac{3i}{4} \sum_{A=4}^{7} (\omega^A)^2 \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(q) D_{\kappa\lambda}(q) \times \left\{ \left[ \Pi_4^{\mu\kappa}(q) - \Pi_1^{\mu\kappa}(q) \right] \left[ \Pi_4^{\nu\lambda}(q) - \Pi_1^{\nu\lambda}(q) \right] + 2\Pi_4^{\mu\kappa}(q) \Pi_4^{\nu\lambda}(q) + \left[ \Pi_4^{\mu\kappa}(q) - \Pi_8^{\mu\kappa}(q) \right] \left[ \Pi_4^{\nu\lambda}(q) - \Pi_8^{\nu\lambda}(q) \right] \right\}, \]

where \( \Pi_A^{\nu\lambda}(q) \) \((A = 1, 4, 8)\) are different components of the gluon polarization tensor.

• Masses of the doublet and antidoublet:

\[ M_{\chi, \bar{\chi}}^2 \simeq C_\chi \sqrt{\alpha} |\Delta|^2, \quad \text{where} \quad C_\chi \simeq 7.5 \]

• Mass of the singlet

\[ M_\eta^2 = 0 \quad \text{(perturbatively)}, \]

• Non-perturbatively (screened instantons)

[Son, Stephanov & Zhitnitsky,'00],

\[ M_\eta^2 \simeq \frac{C_\eta}{\alpha^7} |\Delta^-|^2 \exp \left( -\frac{2\pi}{\alpha} \right), \]

where \( C_\eta \simeq 10^5 \)
**Binding of (anti-) doublets**

- **SU(2)$_c$** is confined at scale \([\text{Rischke, Son & Stephanov, '00}]\)

  \[ \Lambda'_{QCD} \approx |\Delta| \exp \left[ -C_0 \alpha^2 e^{\Delta / \sqrt{\alpha}} \right] \ll M_{\lambda,\chi}, \]

  where \(C_0 \approx 10^{-3}\) and \(C = 3 \left( \frac{\pi}{2} \right)^{3/2}\).

- **Doublet-antidoublet interaction** is

  \[ V(r) \approx -\frac{4\pi\alpha}{\epsilon r}, \quad \text{for} \quad \frac{1}{|\Delta^-|} \ll r \ll \frac{1}{\Lambda'_{QCD}}, \]

  where \(\epsilon \approx \frac{2\alpha\mu^2}{9\pi|\Delta^-|^2}\) is the dielectric constant.

- **Colorless** bound states \((\lambda_0 = \text{udud})\) with binding energy \((\Lambda'_{QCD} \ll E_n \ll M_{\lambda,\chi})\)

  \[ E_n \approx -\frac{M_{\lambda,\chi}}{n^2} \left( \frac{3\pi|\Delta|}{\mu} \right)^4 \sim -\frac{\alpha^{-39/4}}{n^2}|\Delta|e^{-4C / \sqrt{\alpha}} \]

  are formed.
**CFL: Collective modes**

- Current-current correlation function:
  \[
  \langle j_{\mu} j_{\nu} \rangle = \frac{q^2 \Pi_1 O_{\mu\nu}^{(mag)}}{q^2 + \Pi_1} + \frac{q^2 \left[ \Pi_2 \Pi_3 + (\Pi_4)^2 \right] O_{\mu\nu}^{(el)}}{(q^2 + \Pi_2) \Pi_3 + (\Pi_4)^2}
  \]

  where the polarization tensor reads

  \[
  \Pi_{\mu\nu} = \Pi_1 O_{\mu\nu}^{(mag)} + \Pi_2 O_{\mu\nu}^{(el)} + \Pi_3 O_{\mu\nu}^{(||)} + \Pi_4 O_{\mu\lambda}^{(mix)}
  \]

- **NG bosons** (related to chiral symmetry)
  \[
  \omega_{ng} \sim q \quad \text{(for all } T < T_c)\]

- **Plasmon** with mass
  \[
  M_{gl}^{(1)} \sim \omega_p \equiv \frac{g_s \mu}{\sqrt{2\pi}}
  \]

- **“Light” plasmon** with mass
  \[
  1.36 |\Delta| < M_{gl}^{(2)} < 2 |\Delta| \quad \text{(for all } T < T_c)\]

- **Gapless CG modes** (scalars)
  - exist in neacritical region, \( T \in [T^*, T_c] \)
  - look like “revived” NG bosons
  - made of correlated oscillation of superconducting and normal components
**Gapless Carlson-Goldman modes**

- Gapless modes, discovered experimentally in dirty superconductors [Carlson & Goldman, '75]
- Landau damping vs. dirt [Ohashi & Takada, '97]
- Nearcritical region $T \rightarrow T_c - \epsilon$, clean limit,

\[
v^2 + \frac{i}{224T} \frac{45\pi |\Delta|}{24T} v - \frac{9\zeta(3)|\Delta|^2}{8\pi^2T^2} = 0, \quad \frac{|\Delta|}{T} \ll 1
\]
\[
v \equiv \frac{q_0}{|\vec{q}|} = \frac{|\Delta|}{T} (\pm x^* - iy^*), \quad x^* \approx 0.19 \quad y^* \approx 0.32
\]

- Compare with NG bosons (pseudoscalars)

\[
v_{ng}^2 + \frac{i}{32T} \frac{5\pi |\Delta|}{32T} v_{ng} - \frac{7\zeta(3)|\Delta|^2}{8\pi^2T^2} = 0 \quad \frac{|\Delta|}{T} \ll 1
\]
\[
v_{ng} \equiv \frac{q_0}{|\vec{q}|} = \frac{|\Delta|}{T} (\pm x^* - iy^*), \quad x^* \approx 0.22 \quad y^* \approx 0.25
\]
Numerical results (clean limit)

- Spectral densities of electric gluon quasi-particles at $T = 0.9997T_c$ and $T = 0.9999T_c$ (peaks correspond to CG modes, $\omega \sim |\Delta|q$)

- Spectral densities of NG bosons at $T = 0.9997T_c$ and $T = 0.9999T_c$ ($\omega_{ng} \sim |\Delta|q$)
Conclusion and Outlook

- Studies of dense quark matter are under theoretical controle, assuming $\mu \gg \Lambda_{\text{QCD}}$

- Many properties of color superconductors have already been revealed:
  - phases: S2C, CFL, crystalline, gapless
  - color-flavor unlocking ($m_s \neq 0$)
  - (pseudo-)NG boson properties ($\text{CFL, S2C}$)
  - confinement of unbroken $SU(2)_c$ ($\text{S2C}$)
  - collective excitations ($\text{CFL, S2C}$)
  - enforced electrical neutrality ($\text{CFL, } \mu_e, m_s$)
  - sum rules ($\text{CFL}$)
  - neutrino transponrt ($\text{S2C}$)
  - kaon condensation ($\text{CFL, } \mu_e, m_s$)

- Motivation for further studies: color superconductivity is likely to exist in the cores of some compact stars