UnNuclear Physics: Conformal Symmetry in Nuclear Reactions

Dam Thanh Son (University of Chicago)
Igor Shovkovy’s Theoretical Physics Colloquium
November 3, 2021
References

Summary

• Conformal invariance
• Nonrelativistic conformal invariance
• Fermion at unitarity
• Unnuclear physics
• Nuclear reaction with final-state neutrons
Role of symmetry in physics

- Symmetries play a very important role in physics
- In particular, spacetime symmetry is key to understanding of elementary particles
Poincaré symmetry

- 1 time and 3 spatial translations: \( 4 \ P^\mu \ x^\mu \rightarrow x^\mu + a^\mu \)
- 3 rotations + 3 boosts \( M^{\mu\nu} \)

- Elementary particles: irreducible representations of the Poincaré group, characterized by Casimirs:
  - mass and spin when \( m \neq 0 \)
  - when \( m = 0 \): helicity instead of spin
Conformal symmetry

• An extension of Poincaré group: conformal symmetry

• All transformations that preserve angle

• include: dilatation $x^\mu \rightarrow \lambda x^\mu$

• and 4 “proper conformal transformations”

• Field theory with this symmetry: conformal field theory

• applications in theoretical physics including phase transitions
CFT in particle physics?

- The Standard Model is not a conformal field theory
- CFT cannot have massive particles
  - \[ E = \sqrt{p^2 + m^2} \] not invariant under \( E \rightarrow \lambda E, \ p \rightarrow \lambda p \)
- can only have massless particles or some fuzzy “stuff”
Georgi’s unparticle

H. Georgi, 2007

- In CFT: \[ \langle \mathcal{U}(x)\mathcal{U}(0) \rangle = \frac{c}{|x|^{2\Delta_{\mathcal{U}}}} \]
- In momentum space \( G_{\mathcal{U}}(p) \sim p^{2\Delta_{\mathcal{U}}-4} \)
- Particle: \( \Delta_\phi = 1, \ G_\phi(p) \sim p^{-2} \)
- but otherwise the propagator has cuts, not poles
- Energy is not fixed when momentum is fixed: \( E > pc \)
- Georgi: unparticle, hypothesize that it can be a hidden sector coupled to the SM
Signal of unparticles

- Energy spectrum of $B$ is **continuous**
- near end point depends on the dimension of $\mathcal{U}$:

$$\frac{d\sigma}{dP^2_{\mathcal{U}}} \sim (P^2_{\mathcal{U}})^{\Delta-2}$$
CMS collaboration: no unparticle found so far at LHC

Search for unparticles

CMS: "95% confidence limits are obtained on the effective cut-off scale as a function of the scaling dimension"
• **Nonrelativistic** conformal field theory is fruitful
• Realized in nature
• has experimentally verifiable consequences
Schrödinger symmetry

- Nonrelativistic version of conformal symmetry: “Schrödinger symmetry”
- Symmetry of the free time-dependent Schrödinger equation

\[ i \partial_t \psi = -\frac{\nabla^2}{2m} \psi \]

Galilean boost: \( \tilde{\psi}(t, x) = e^{imv \cdot x - \frac{i}{2}mv^2t} \psi(t, x - vt) \)

Dilatation: \( \tilde{\psi}(t, x) = \psi(\lambda^2 t, \lambda x) \)

Special conformal transformation:

\[ \tilde{\psi}(t, x) = \frac{1}{(1 + \alpha t)^{3/2}} \exp \left( \frac{i}{2 \ 1 + \alpha t} \right) \psi \left( \frac{t}{1 + \alpha t}, \frac{x}{1 + \alpha t} \right) \]
Schrödinger algebra

- Free particles \((\mathbf{x}_a, \mathbf{p}_a), a = 1, 2, \ldots N\)
  
  \[
  P = \sum_a p_a \quad H = \sum_a \frac{p_a^2}{2m}
  \]

- Momentum \(K = \sum mx_a\) Galilean boosts

- Kinetic energy \(D = \sum \frac{1}{2}(\mathbf{x}_a \cdot \mathbf{p}_a + \mathbf{p}_a \cdot \mathbf{x}_a)\) dilatation

- Angular momentum

- Mass \(M = Nm\)
Schrödinger algebra

\[
\begin{array}{|c|cccccc|}
\hline
X \backslash Y & P_j & K_j & D & C & H \\
\hline
P_i & 0 & -i\delta_{ij}M & -i P_i & -i K_i & 0 \\
K_i & i\delta_{ij}M & 0 & i K_i & 0 & i P_i \\
D & i P_j & -i K_j & 0 & -2i C & 2i H \\
C & i K_j & 0 & 2i C & 0 & i D \\
H & 0 & -i P_j & -2i H & -i D & 0 \\
\hline
\end{array}
\]

\[
[J_{ij}, N] = [J_{ij}, D] = [J_{ij}, C] = [J_{ij}, H] = 0, \\
[J_{ij}, P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i), \quad [J_{ij}, K_k] = i(\delta_{ik}K_j - \delta_{jk}K_i), \\
[J_{ij}, J_{kl}] = i(\delta_{ik}J_{jl} + \delta_{jl}J_{ik} - \delta_{il}J_{jk} - \delta_{jk}J_{il}).
\]
Special role of dilatation

\[ D = \sum \frac{1}{2}(x_a \cdot p_a + p_a \cdot x_a) \]

- Dilatation operator rescales coordinates and momenta:
  - \[ p \rightarrow \lambda p, x \rightarrow \lambda^{-1}x \]
  - \[ [D, P_i] = iP_i \]
  - \[ H = p^2/2m \rightarrow \lambda^2 H \]
  - \[ [D, H] = 2iH \]
Beyond free theory

- Is the Schrödinger symmetry good only for non-interacting theory?
- Most general Hamiltonian: not scale-invariant $[D, H] \neq 2iH$
- There exists a way to have the symmetry in interacting theory: **unitarity regime**
Unitarity regime

- Take a potential of a certain shape (e.g., square well)
- shrink the range, adjusting the depth so that there is one bound state at threshold
- In the language of scattering theory: infinite scattering length, zero range
- Interaction has no energy/length scale: Hamiltonian is scale invariant \([D, H] = 2iH\)
Properties of unitary gas

- A gas of spin-1/2 particles with fine-tuned to unitarity
- Can be realized with trapped cold atoms Feshbach resonance
- Scale invariance: physical quantities can be figured out by scaling arguments
- Example: Bertsch parameter $\xi$ ($T = 0$)

$$\frac{E}{N} = \xi \frac{3}{5} \varepsilon_F, \quad \varepsilon_F = \frac{1}{2m} (3\pi^2 n)^{2/3}$$
Shear and bulk viscosities

- Scaling: \( \eta, \zeta = \hbar n f_{\eta,\zeta} \left( \frac{T}{\varepsilon_F} \right) \)
- Conformal invariance: \( \zeta = 0 \)
Nonrelativistic CFT

• One can build up the formalism of nonrelativistic conformal field theory in analogy with the relativistic theory

• Many notions can be extended
  • primary operators
  • operator-state correspondence
Fermions at unitarity as a NRCFT

- \[ L = i \psi^\dagger \left( \partial_t + \frac{\nabla^2}{2m} \right) \psi - c_0 \psi^\dagger \psi^\dagger \psi \psi \dagger \quad \Delta[\psi] = \frac{3}{2} \]

- Introducing auxiliary field \( \phi \)

- \[ L = i \psi^\dagger \left( \partial_t + \frac{\nabla^2}{2m} \right) \psi - \psi^\dagger \psi^\dagger \phi - \phi^\dagger \psi \psi \dagger + \frac{\phi^\dagger \phi}{c_0} \]

- Propagator of \( \phi \)

\[ G_\phi(\omega, p) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}} \quad \Delta[\phi] = 2 \neq 2 \times \frac{3}{2} \]
Renormalization

- \( G_{\phi}^{-1}(\omega, p) = c_0^{-1} + \text{one-loop integral} \)

- \( = c_0^{-1} + \Lambda + \left( \frac{p^2}{4m} - \omega \right)^{1/2} \)

- Unitarity: fine-tuning so that \( c_0 + \Lambda = 0 \)

- (scattering length: \( c_0 + \Lambda = \frac{1}{a} \))

- Physically: fine-tune the attractive short-range potential to have a bound state at threshold

\[
G_{\phi}(\omega, p) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}
\]
Local operators

- Local operators are classified by mass and dimension
  - \([M, O(x)] = -M_o O(x)\)
  - \([D, O(0)] = i\Delta O(0)\)
- Commuting with \(P\) and \(H\) increases the dimension by 1 and 2, commuting with \(K\) and \(C\) by \(-1\) and \(-2\)
- Representation theory for operators with \(M \neq 0\) is simple
Raising and lowering dimensions

- Operators with $M \neq 0$ are organized in towers
- Dimension raised by $P$ and $H$, lowered by $K$ and $C$
- Primary operators: lowest in a ladder

\[
[K, O(0)] = [C, O(0)] = 0
\]
• Dimension of a primary operator = energy of a state in a harmonic potential

• Example: in free theory $|\psi| = \frac{d}{2}$, ground state of 1 particle in harmonic potential: $E = \frac{d}{2} \hbar \omega$
Two-point function of a primary operator

\[ G_{\mathcal{U}}(t, x) = -i\langle T\mathcal{U}(t, x)\mathcal{U}^\dagger(0, 0) \rangle = C \frac{\theta(t)}{(it)^\Delta} \exp\left( \frac{iMx^2}{2t} \right) \]

\[ G_{\mathcal{U}}(\omega, p) \sim \left( \frac{p^2}{2M} - \omega \right)^{\Delta - \frac{5}{2}} \]

\[ \omega - \frac{p^2}{2M} \] is the energy in the CM frame
Operator dimensions for fermions at unitarity

- Dimensions of operators: either by field theory or quantum mechanical calculation in a harmonic trap
- Lowest-dimension operators

<table>
<thead>
<tr>
<th>$N$</th>
<th>$S$</th>
<th>$L$</th>
<th>$O$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>$\psi_\uparrow \psi_\downarrow$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1/2</td>
<td>1</td>
<td>$\psi_\downarrow \psi_\uparrow \nabla \psi_\uparrow$</td>
<td>4.273</td>
</tr>
<tr>
<td>3</td>
<td>1/2</td>
<td>0</td>
<td>$\psi_\downarrow \nabla \psi_\uparrow \cdot \nabla \psi_\uparrow$</td>
<td>4.666</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>$\psi_\downarrow \psi_\uparrow \nabla \psi_\downarrow \cdot \nabla \psi_\uparrow$</td>
<td>5.0–5.1</td>
</tr>
</tbody>
</table>
Y. Nishida, DTS, arXiv:1004.3597
“UnNuclear physics”

A nonrelativistic version of unparticle physics field in NRCFT: “unnnucleus”

H.-W. Hammer and DTS, 2103.12610
Few-neutron systems as unnuclei

- Neutrons have anomalously large scattering length: \( a_{nn} \approx -19 \text{ fm} \)
- vs effective range \( r_0 \approx 2.8 \text{ fm} \)
- In a wide range of energy is neutrons are fermions at unitarity
Nuclear reactions

• Many nuclear reactions with emissions of neutrons:
  • $^3\text{H} + ^3\text{H} \rightarrow ^4\text{He} + 2\text{n}$
  • $^7\text{Li} + ^7\text{Li} \rightarrow ^{11}\text{C} + 3\text{n}$
  • $^4\text{He} + ^8\text{He} \rightarrow ^8\text{Be} + 4\text{n}$

• Final-state neutrons can be considered as forming an “unnucleus” - a field in NRCFT

• Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/m a^2 \sim 0.1$ MeV
  $\hbar^2/m r_0^2 \sim 5$ MeV
Few-neutron systems as unnuclei

In the regime $E_0 < E \ll E_0$, ignoring the energy dependence of all other factors, we can write

$$d \sigma \sim \left| \mathcal{M} \right|^2 \sqrt{E_B} \times \text{Im} G_{\mathcal{U}}(E_\mathcal{U}, \mathbf{p})$$

primary reaction has larger energy than final-state interaction
Rates of processes involving an unnucleus

\[ \frac{d\sigma}{dE} \sim |\mathcal{M}|^2 \sqrt{E} \times \text{Im} G_U(E_{\text{kin}} - E, \vec{p}) \]

\[ \left( E_{\text{kin}} - E - \frac{p^2}{2M_U} \right)^{\Delta - \frac{5}{2}} \]

Near end point: \[ \frac{d\sigma}{dE} \sim (E_0 - E)^{\Delta - \frac{5}{2}} \]
Nuclear reactions

- $^3\text{H} + ^3\text{H} \rightarrow ^4\text{He} + 2\text{n}$  
  $\alpha = -0.5$
- $^7\text{Li} + ^7\text{Li} \rightarrow ^{11}\text{C} + 3\text{n}$  
  $\alpha = 1.77$
- $^4\text{He} + ^8\text{He} \rightarrow ^{8}\text{Be} + 4\text{n}$  
  $\alpha = 2.5 - 2.6$

Prediction:

$$\frac{d\sigma}{dE} \sim (E_0 - E)^\alpha$$

Regime of validity: kinetic energy of neutrons in their c.o.m. frame between

- $\hbar^2/ma^2 \sim 0.1$ MeV
- $\hbar^2/mr_0^2 \sim 5$ MeV
Comparison with microscopic models

\[ \pi^- + ^3H \rightarrow \gamma + 3n \]
FIG. 4. Center-of-mass energy spectrum of three neutrons in the reaction \(3^\text{H}(\mu^-,\nu_\mu)3n\) (left panel) and \(3^\text{H}(\mu,\nu_\mu)3n\) (right panel). The circles/squares give the full/plane wave calculations by Golak et al. \([23,24]\). Different fits are explained in the legend and in the main text. The calculated photon spectra to three-neutron spectra for convenience. As expected, the free neutron behavior, \(E_{3n}(\text{dashed line})\), can describe the full calculation (circles) only at the lowest energies. However, the plane wave impulse approximation (squares) can be described up to about 2.5 MeV. The full calculation including final state interaction displays clear unnucleus behavior, \(E_{1.77}\) (solid line) for energies also up to about 2.5 MeV, where it starts to deviate from the prediction. This is somewhat smaller than the value 5 MeV expected from the scattering length. We suspect that this is due to the wave function of the triton, which has finite extent, making the reaction a less than ideal “point source” of the neutrons and causing the factorization formula \((13)\) to be realized nearer than expected. The description cannot be significantly improved by including the next state which behaves as \(E_{2.17}\) (dash-dotted line). Analogous behavior is exhibited by the theoretical spectra for the reaction \(3^\text{H}(\mu,\nu_\mu)3n\) calculated by Golak et al. \([24]\) using the same interaction model (see right panel of Fig. 4). In this reaction, the energy scale of the primary scattering process is slightly smaller such that the corrections to factorization are larger.

A four-neutron spectrum was recently measured by Kisamori et al. in the reaction \(4^\text{He}(8^\text{He},8^\text{Be})4^\text{n}\) \([25]\), but the number of events is too low to extract evidence of unnucleus behavior. It may, however, be possible to extract such behavior from the spectra of a new experiment using the reaction \(8^\text{He}(p,p\nu_\mu)4^\text{n}\), which are currently being analyzed \([18]\).
Conclusion

• There is a nonrelativistic version of conformal field theory
• Example: fermions at unitarity
• Approximately realized by neutrons; leads to “unnuclear behavior”
• Possible extension to other systems
  \(X(3872)\) Braaten and Hammer
Thank you