Einstein-Cartan gravity: Inflation, Dark Matter and Electroweak Symmetry Breaking

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Based on:

MS, Andrey Shkerin, and Sebastian Zell:

- Standard Model Meets Gravity: Electroweak Symmetry Breaking and Inflation, 2001.09088

- Quantum Effects in Palatini Higgs Inflation, 2002.07105

MS, Andrey Shkerin, Inar Timiryasov and Sebastian Zell:

- Einstein-Cartan gravity, matter, and scale-invariant generalisation, 2007.16158

- Higgs inflation in Einstein-Cartan gravity, 2007.14978

- Einstein-Cartan Portal to Dark Matter, 2008.11686

Outline

- Metric, Palatini and Einstein-Cartan gravities
- Bosonic action in EC gravity with Higgs field
- Inflation
- Fermion action in EC gravity: Einstein-Cartan portal to dark matter
- Non-perturbative generation of the electroweak scale
- Conclusions

Short reminder

Flat Minkowski space-time in arbitrary coordinates $x^\mu = f^\mu(\xi^i)$ (ξ^i - Cartesian coordinates). Metric $g_{\mu\nu}(x)$ and connection $\Gamma^\alpha_{\mu\nu}$ (describing the parallel transport of a vector and covariant derivatives), $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$ can be found from coordinate transformation $x^\mu = f^\mu(\xi^i)$ and have the following properties:

1. invariant (length) interval, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
2. connection is symmetric, $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$
3. $g_{\mu\nu;\alpha} = 0$: metricity - length of a vector is constant at the parallel transport
4. Covariant derivatives commute, $\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu = 0$

Geometric approach to gravity, Riemann geometry

- distances: symmetric metric tensor
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$; symmetric connection $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$.
  Same as 2

- metricity, local Minkowski structure:
  $g_{\mu\nu;\alpha} = 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$. Same as 3.

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu} V_\beta$. Different from 4!

Geometric approach to gravity, Cartan geometry, 1922-1925

- distances: symmetric metric tensor
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$; arbitrary connection $\Gamma^\alpha_{\mu\nu} \neq \Gamma^\alpha_{\nu\mu}$. New object: torsion tensor $T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}$. Different from 2!

- metricity, local Minkowski structure (same as 3):
  $g_{\mu\nu;\alpha} = 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$ and torsion $T^\alpha_{\mu\nu}$

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu} V_\beta$. Different from 4!

Geometric approach to gravity, non-metricity

- distances: symmetric metric tensor $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha}V^\nu dx^\alpha$; arbitrary connection $\Gamma^\alpha_{\mu\nu} \neq \Gamma^\alpha_{\nu\mu}$. New object: torsion tensor $T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}$. Different from 2!

- non-metricity. New object: non-metricity tensor $g_{\mu\nu;\alpha} = Q_{\mu\nu\alpha} \neq 0$
  $\longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$, torsion $T^\alpha_{\mu\nu}$ and non-metricity tensor $Q_{\mu\nu\alpha}$. Length of the vectors changes with parallel transport. Different from 3!

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu}V_\beta$. Different from 4!

Geometric approach to gravity, Weyl theory (1918)

- distances: symmetric metric tensor $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Same as 1.

- Symmetric connection $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$. Same as 2.

- non-metricity. New object - vector field A_μ: non-metricity tensor is reduced to $g_{\mu\nu;\alpha} = Q_{\mu\nu\alpha} = -2A_\alpha g_{\mu\nu} \neq 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$ and vector field A_μ. Different from 3!

- Commutator of covariant derivatives: Riemann tensor $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu}V_\beta$. Different from 4!

Dynamics, Einstein-Hilbert metric action (1915)

- Lowest order action (without cosmological constant) is

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}\,R$$

- The dynamical variable is $g_{\mu\nu}$, variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations. (We use mostly positive metric.)

Dynamics, Palatini action (1919)

Palatini gravity

Basic structures: metric $g_{\mu\nu}$ (distances) and symmetric connection $\Gamma^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\sigma\nu}$. Riemann curvature tensor is expressed via connection as:

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Same as in metric gravity

Lowest order action (without cosmological constant) is

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}\,R$$

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives metricity $g_{\mu\nu;\alpha} = 0$, i.e. the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations.

Palatini pure gravity is equivalent to metric gravity

Metric, Palatini and Einstein-Cartan gravities

Einstein-Cartan gravity

Basic structures: metric $g_{\mu\nu}$ (distances) and connection $\Gamma^{\rho}_{\nu\sigma} \neq \Gamma^{\rho}_{\sigma\nu}$. Riemann curvature tensor is expressed via connection as:

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Same as in metric gravity

New object - torsion tensor: $T^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\nu\sigma} - \Gamma^{\rho}_{\sigma\nu}$

Lowest order action (without cosmological constant) is

Holst term

Same as in metric gravity

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R + \frac{M_P^2}{2\gamma}\int \mathrm{d}^4x\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

Barbero-Immirzi parameter

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations. On the solution $g_{\mu\nu;\alpha} = 0$ and $T^{\rho}_{\nu\sigma} = 0$.

Einstein-Cartan pure gravity is equivalent to metric gravity

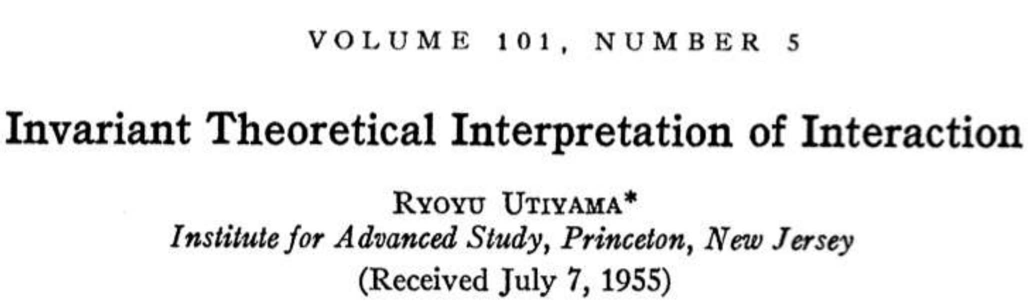
EC gravity as a gauge theory

Existence of electromagnetic field - U(1) global invariance of fermion Lagrangian promoted to be local

Gluons, W^+, W^- Z and γ of the Standard Model - SU(3)xSU(2)xU(1) global invariance of SM fermion Lagrangian promoted to be local

Existence of gravitational field - Poincare invariance of SM fermion Lagrangian promoted to be local?

Gauging of the Poincare group: Utiyama (1956), Kibble (1961) , Sciama (1962,1964).



VOLUME 101, NUMBER 5

Invariant Theoretical Interpretation of Interaction

Ryoyu Utiyama*
Institute for Advanced Study, Princeton, New Jersey
(Received July 7, 1955)

EC gravity as a gauge theory

Basic gauge fields

- e^I - tetrad one-form (frame field, translations), I=0,1,2,3
- ω^{IJ} - spin connection one form (gauge field of the local Lorentz group).
  Euclidean: $SO(4) \sim SU(2)_L \times SU(2)_R$
- $F^{IJ} = d\omega^{IJ} + \omega^I_K \omega^{KJ}$: curvature 2-form

Pure gauge action:

$$\frac{M_P^2}{4}\int \epsilon_{IJKL} e^I e^J F^{KL} + \frac{M_P^2}{2\gamma}\int e^I e^J F_{IJ}$$

Again, equivalent to metric gravity

EC gravity with matter fields

Once matter fields are added, the equivalence between different formulation of gravity is lost:

Couplings to

- scalar fields: $\phi^2 \epsilon_{IJKL} e^I e^J F^{KL}$ (or $\phi^2 R$)
- and to fermion fields via covariant derivative $D\Psi = d\Psi + \frac{1}{8}\omega_{IJ}[\gamma^I, \gamma^J]\Psi$

lead to modified relation between the spin connection and tetrad (or Christoffel symbols and the metric). Torsion is non-zero.

Physics is different!

Bosonic action in EC gravity with Higgs field

Inclusion of the scalar field (Higgs field of the Standard Model, unitary gauge)

Scalar action

$$S_h = \int d^4x \sqrt{-g}\left(-\frac{1}{2}\left(\partial_\mu h\right)^2 - U(h)\right), \quad U(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

Gravity part

Same as in metric gravity

$$S_{\text{grav}} = \frac{1}{2}\int d^4x \sqrt{-g}\left(M_P^2 + \xi h^2\right)R$$

Holst term

$$+ \frac{1}{2\bar{\gamma}}\int d^4x \sqrt{-g}\left(M_P^2 + \xi_\gamma h^2\right)\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

Nieh-Yan invariant

$$+ \frac{1}{2}\int d^4x\, \xi_\eta h^2 \partial_\mu\left(\sqrt{-g}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

Three non-minimal couplings:
$\xi,\ \xi_\gamma,\ \xi_\eta$

For $1/\bar{\gamma} = \xi_\gamma = \xi_\eta = 0$ we get the Palatini action with non-minimal coupling.

Bosonic action in EC gravity with Higgs field

- Torsion is not dynamical
- Same number of degrees of freedom as in the metric gravity + scalar field : 2 (graviton) +1 (scalar)
- Equivalent metric theory : use the Weyl transformation of the metric field

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Modified kinetic term:
essential for non-perturbative generation of the electroweak scale and inflation

Metric action:

$$S_{\text{metric}} = \frac{M_P^2}{2} \int d^4x \sqrt{|g|} \left\{ R - \left[\frac{1}{2\Omega^2} (\partial_\mu h)^2 + \frac{U}{\Omega^4} \right] - \frac{3M_P^2}{4(\gamma^2+1)} \left(\frac{\partial_\mu \bar{\eta}}{\Omega^2} + \partial_\mu \gamma \right)^2 \right\}$$

$$\gamma = \frac{1}{\bar{\gamma}\Omega^2} \left(1 + \frac{\xi_\gamma h^2}{M_P^2} \right), \quad \bar{\eta} = \frac{\xi_\eta h^2}{M_P^2}$$

Flat potential:
essential for inflation

Inflation

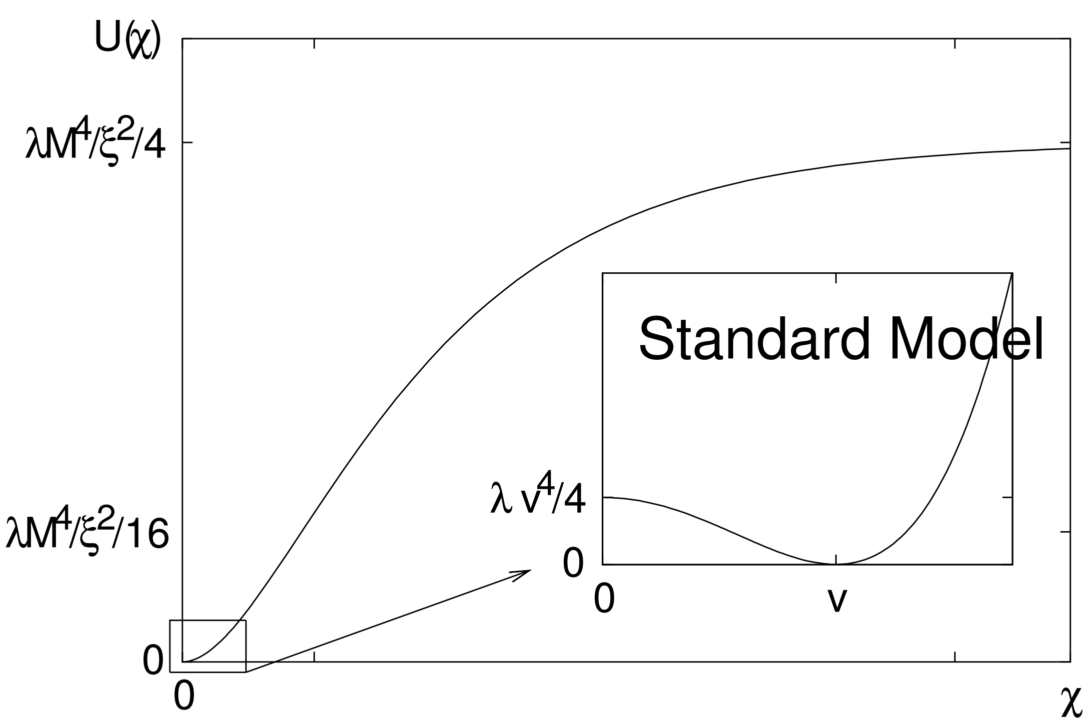
Modified kinetic term:
essential for non-perturbative generation
of the electroweak scale and inflation

Metric action:

$$S_{\rm metric} = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}\left\{R - \left[\frac{1}{2\Omega^2}(\partial_\mu h)^2 + \frac{U}{\Omega^4}\right] - \frac{3M_P^2}{4(\gamma^2+1)}\left(\frac{\partial_\mu \bar\eta}{\Omega^2} + \partial_\mu\gamma\right)^2\right\}$$

$$\gamma = \frac{1}{\bar\gamma\Omega^2}\left(1 + \frac{\xi_\gamma h^2}{M_P^2}\right), \quad \bar\eta = \frac{\xi_\eta h^2}{M_P^2}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Flat potential:
essential for inflation



Metric Higgs inflation (Bezrukov, MS):
in limit of the vanishing Holst term,
$\bar\gamma \to \infty, \quad \xi_\gamma = 0,$ take $\quad \xi_\eta = \xi.$

Palatini Higgs inflation (Bauer, Demir):
in limit of the vanishing Holst term,
$\bar\gamma \to \infty, \quad \xi_\gamma = 0,$ take $\quad \xi_\eta = 0$

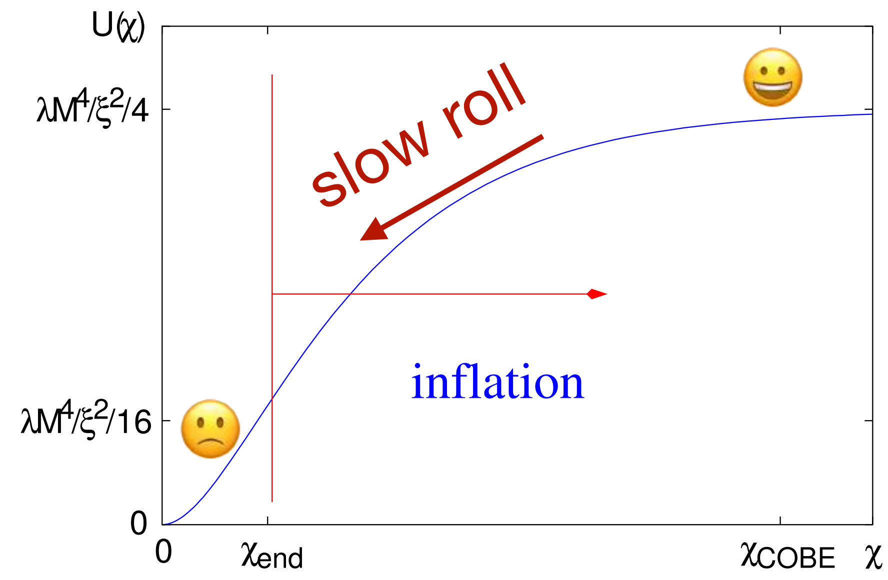
Stages of Higgs Inflation

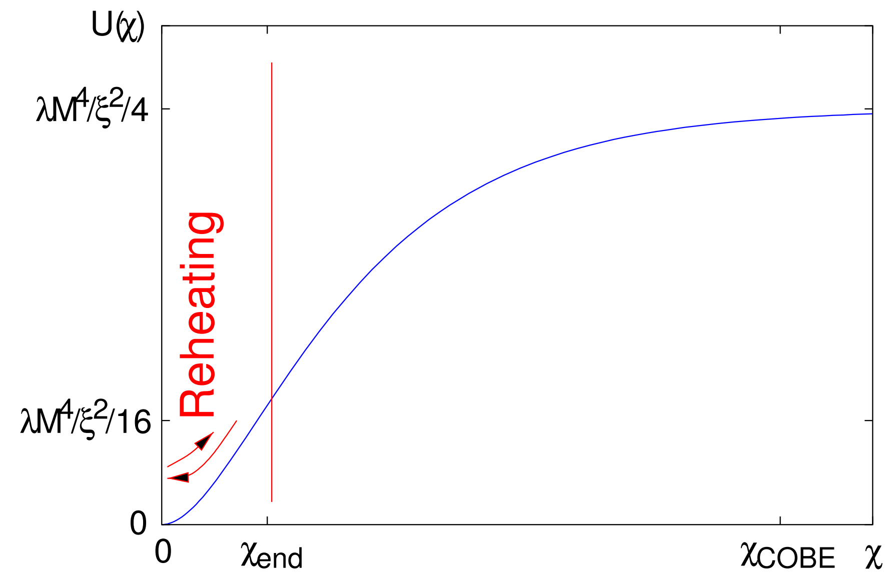
- Chaotic initial conditions: large fields on the plateau inflate, the small fields do not

- Slow roll making the universe flat, homogeneous and isotropic, and producing fluctuations leading to structure formation: clusters of galaxies, etc

- Heating of the Universe : energy stored in the Higgs field goes into the particles of the Standard Model - Higgs makes the Big Bang

- Radiation dominated stage of the Universe expansion starts, leading to baryogenesis, dark matter production, nucleosynthesis…





Metric and Palatini Higgs inflations

$$S_{\text{grav.+h.}} = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{1}{2}K(h)\left(\partial_\mu h\right)^2 - \frac{\lambda}{4\Omega^4}h^4 \right\}$$

$$\frac{dh}{d\chi} = \frac{1}{\sqrt{K(h)}}$$

$$S_{\text{grav.+h.}} = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{1}{2}\left(\partial_\mu \chi\right)^2 - U(\chi) \right\}$$

$$U(\chi) = \frac{\lambda}{4}F(\chi)^4$$

UV cutoff at $h = 0$: $\Lambda \sim M_P/\xi$

UV cutoff at $h = 0$: $\Lambda \sim M_P/\sqrt{\xi}$

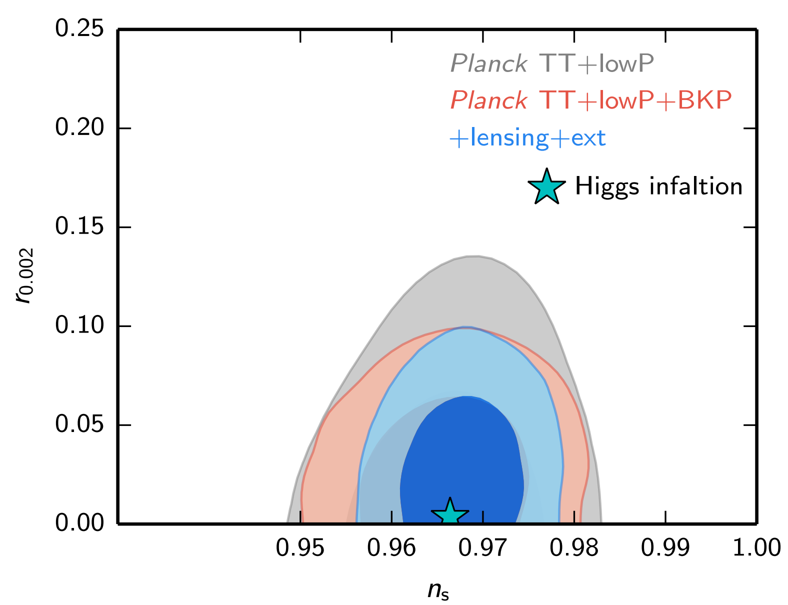
	Metric		Palatini
$K(h)$	$\frac{1}{\Omega^2} + \frac{6\xi^2 h^2}{M_P^2\Omega^4}$		$\frac{1}{\Omega^2}$
$h(\chi)$	$\frac{M_P}{\sqrt{\xi}}\exp\frac{\chi}{\sqrt{6}M_P}$	for $h \gg \frac{M_P}{\sqrt{\xi}}$	$\frac{M_P}{\sqrt{\xi}}\sinh\frac{\sqrt{\xi}\chi}{M_P}$
$F(\chi)$	$\begin{cases} \chi & \chi < \frac{M_P}{\xi} \\ \sqrt{\frac{\sqrt{2}M_P\chi}{\sqrt{3}\xi}} & \frac{M_P}{\xi} < \chi < M_P \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & M_P < \chi \end{cases}$		$\frac{M_P}{\sqrt{\xi}}\tanh\frac{\sqrt{\xi}\chi}{M_P}$
T_{reh}	$3\cdot 10^{15}$ GeV		$\approx 4\cdot 10^{13}$ GeV
N	55.4		$54.9 - \frac{1}{4}\ln\xi \approx 50.9$
n_s	$1 - \frac{2}{N} = 0.964$		$1 - \frac{2}{N} \approx 0.961$
r	$\frac{12}{N^2} = 3.9\cdot 10^{-3}$		$\frac{2}{\xi N^2} \approx 7.7\cdot 10^{-11}$

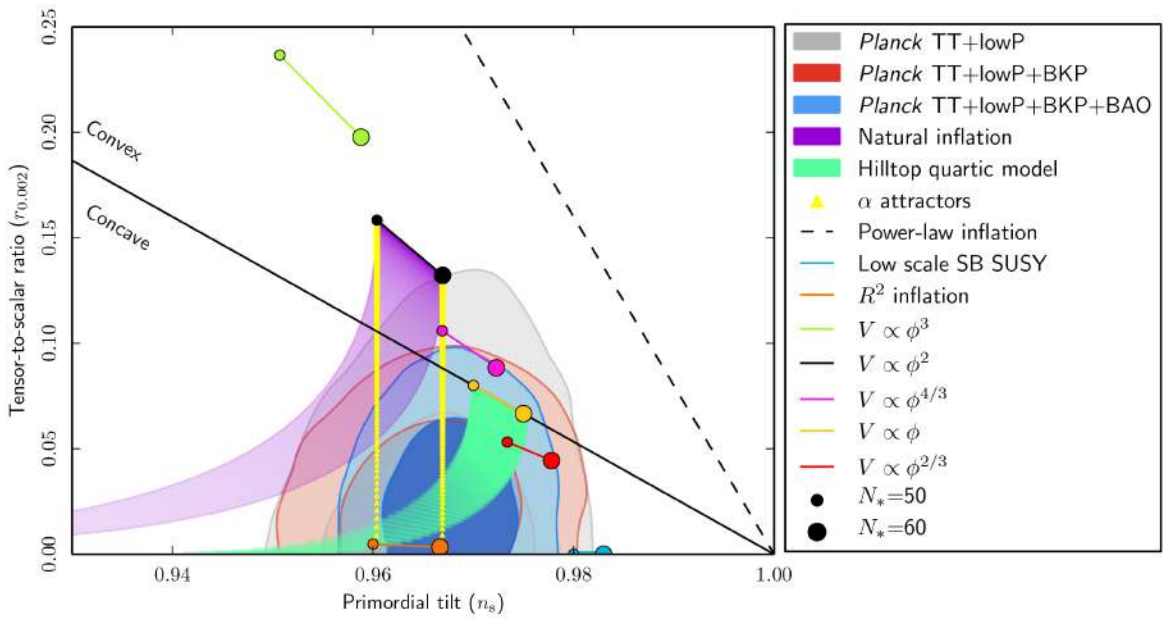
Table 1: Comparison between metric and Palatini Higgs inflation at tree level and for $\xi \gg 1$. In the Palatini case, we use $\xi = 10^7$. Since the analysis of this paper shows that ξ can deviate from this value by an order of magnitude, we use the symbol "$\approx$" when displaying numerical values that depend on ξ. Expressions for n_s and r are given to the leading order in N^{-1} and ξ^{-1}.

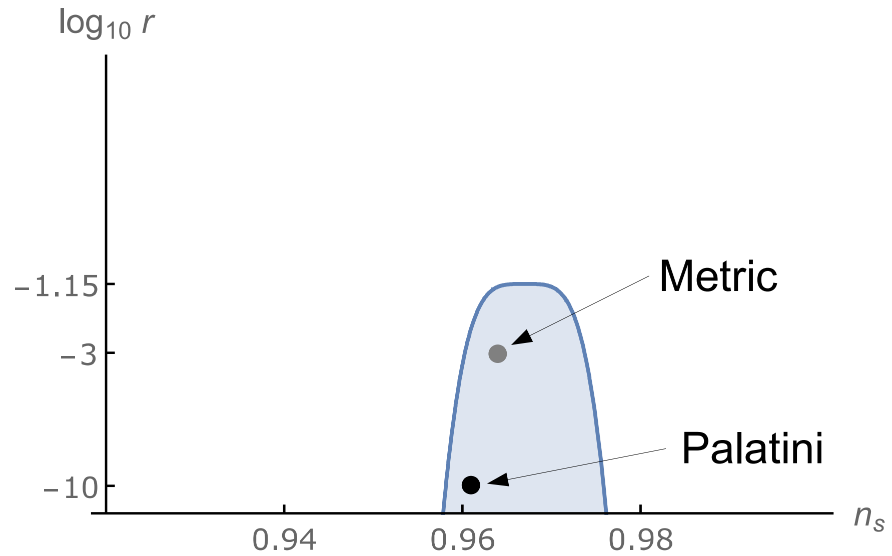
Advantages of the Palatini formulation:

- Parametrically larger UV cutoff
- More robust relation between low energy and high energy parameters
- Natural relation between Fermi and Planck scales

Predictions of metric and Palatini Higgs inflations







Generic EC Higgs inflation

By varying continuously the Higgs coupling to the Nieh-Yan term, one can deform one model into another!

New types of the Higgs inflation (see also Langvik, Ojanpera, Raatikainen and Rasanen, 2007.12595):

- "Nieh-Yan Higgs inflation": vanishing Holst term, $\bar{\gamma} \to \infty, \quad \xi_\gamma = 0.$
- "Holst Higgs inflation", $\xi_\gamma = \xi_\eta = 0.$
- Generic Einstein-Cartan Higgs inflation.

Nieh-Yan Higgs inflation

Vanishing Holst term, $\bar{\gamma} \to \infty, \;\; \xi_\gamma = 0$

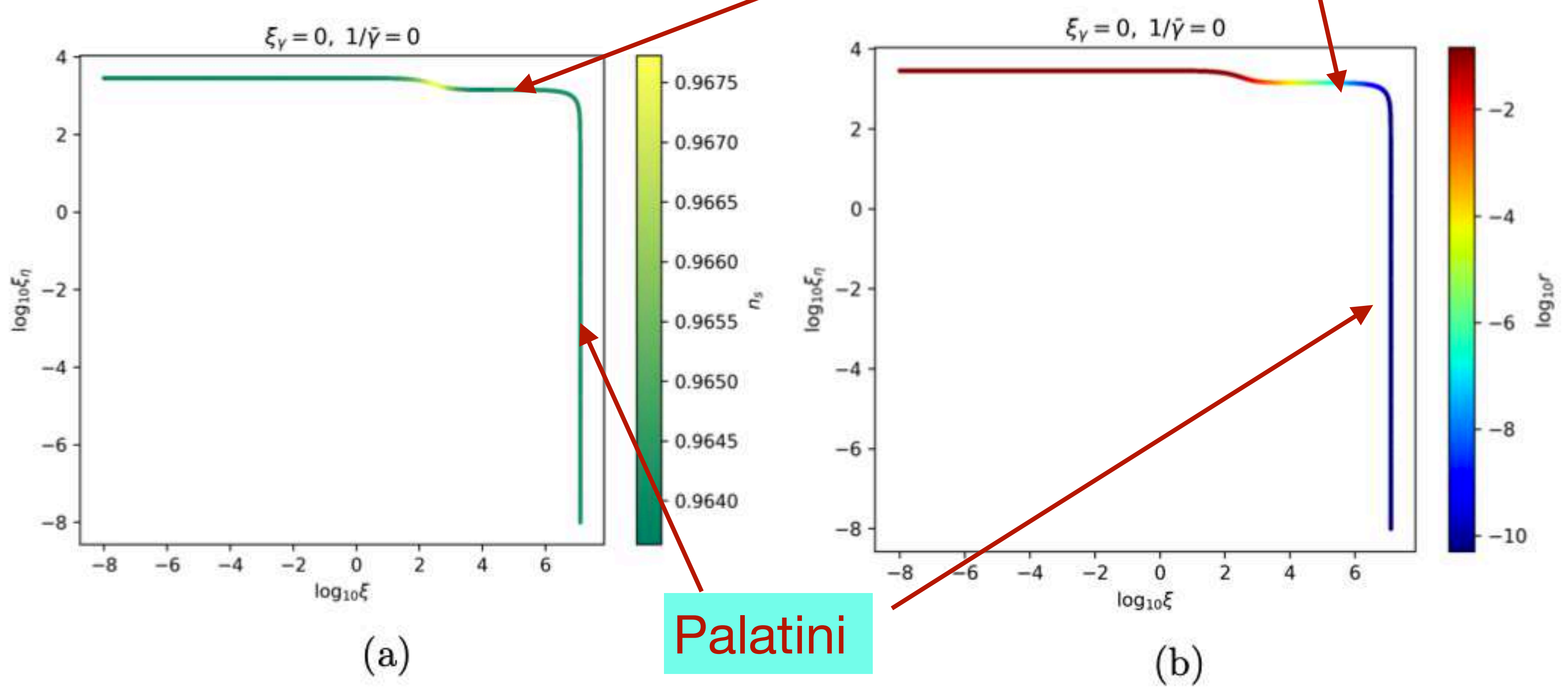
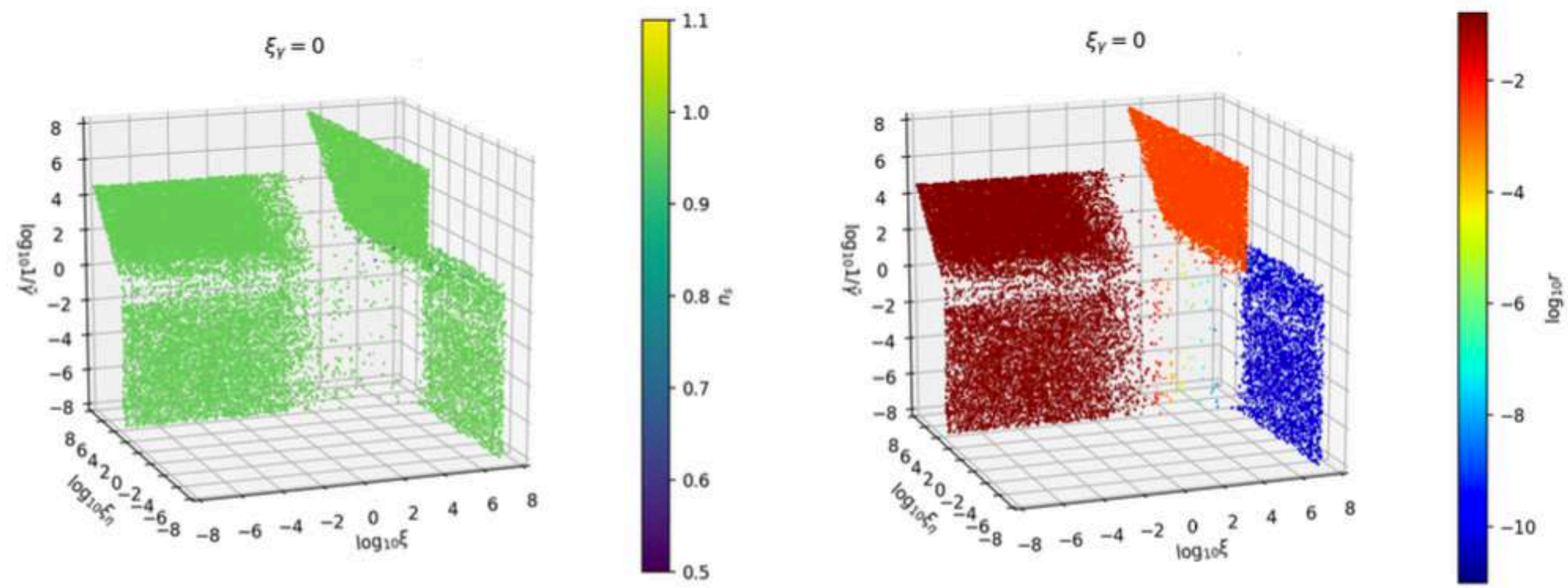


Figure 1. Spectral tilt (a) and tensor-to-scalar ratio (b) in Nieh-Yan inflation. We take $N_\star = 55$ and $\lambda = 10^{-3}$. The regions of Palatini (the right vertical segment) and metric (the "ankle" at which n_s and r vary considerably) Higgs inflation are clearly distinguishable. The transition between the two regions is smooth and stays within the observational bounds. The left horizontal segment has $r > 0.1$ and is not compatible with observations.

Generic Einstein-Cartan Higgs inflation



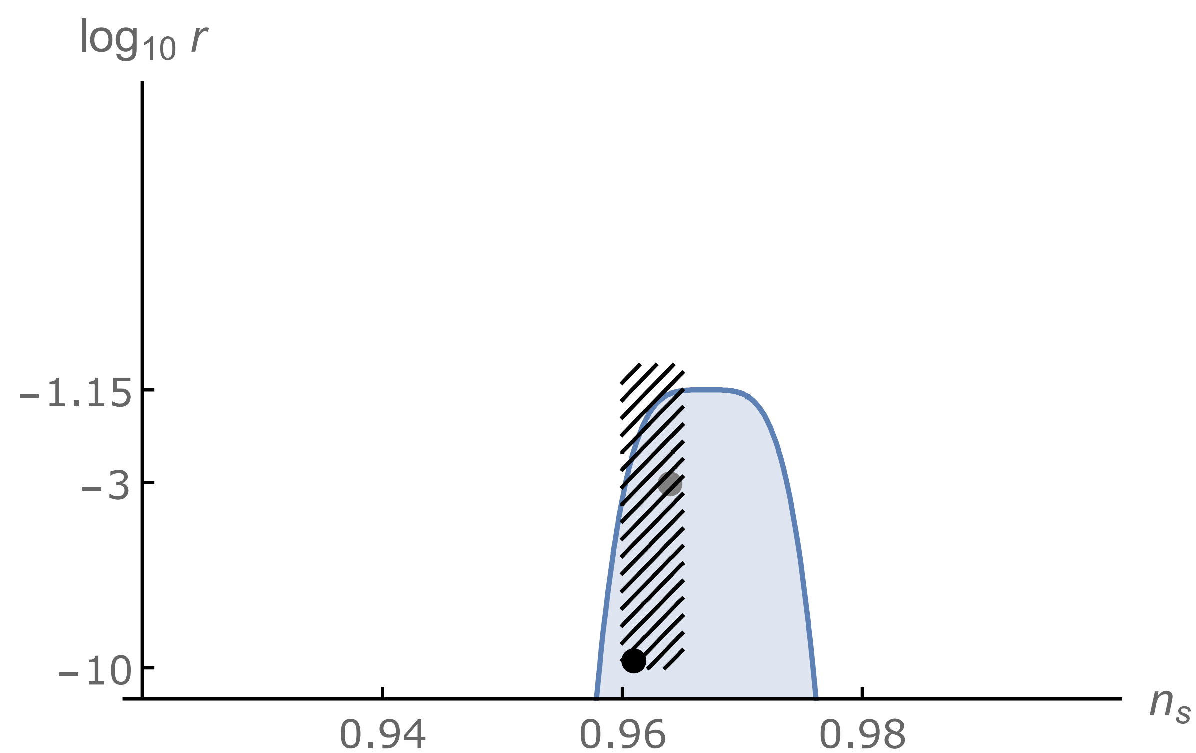
(a) (b)

Figure 5. Spectral tilt (a) and tensor-to-scalar ratio (b) in the case $\xi_\gamma = 0$. One can see that two regions in the right part of the plots reproduce metric and Palatini Higgs inflation. The left region is completely new. Note that due to the large values of the tensor-to-scalar ratio, this region is observationally excluded.

Generic Einstein-Cartan Higgs inflation

Observations:

- Inflation is a generic phenomenon.
- Large parts of the parameter space reproduce the predictions of either metric or Palatini Higgs inflation.
- The spectral index n_s is mostly independent of the choice of couplings and lies very close to $n_s = 1 - 2/N$.
- The tensor-to-scalar ratio r can vary between 1 and 10^{-10}. Detection of r in near future?



Fermion action in EC gravity and Dark Matter production

Inclusion of fermions

Fermion action:

$$S_f = \frac{i}{12}\int \epsilon_{IJKL} e^I e^J e^K \Big(\bar{\Psi}\big(1 - i\alpha - i\beta\gamma^5\big)\gamma^L D\Psi - \overline{D\Psi}\big(1 + i\alpha + i\beta\gamma^5\big)\gamma^L \Psi \Big)$$

$$D\Psi = d\Psi + \frac{1}{8}\omega_{IJ}[\gamma^I, \gamma^J]\Psi$$

Real parameters α, β are non-minimal fermion couplings. They vanish in the case of zero torsion, but in the general case, they contribute to the interactions between the fermionic currents in the effective metric theory.

Fermion action in EC gravity and Dark Matter production

Integrating out torsion one arrives at new universal four-fermion interaction:

$$\int d^4x\sqrt{-g}\,\frac{3}{16M_P^2\left(\gamma^2+1\right)}\left(\left(1+2\gamma\beta-\beta^2\right)A_\mu^2+2\alpha(\gamma-\beta)A_\mu V^\mu-\alpha^2V_\mu^2\right)$$

Vector current: $V_\mu = \bar{N}\gamma_\mu N + \sum \bar{X}\gamma_\mu X$

New fermion - dark matter particle

All fermions of the SM

Axial current: $A_\mu = \bar{N}\gamma_5\gamma_\mu N + \sum \bar{X}\gamma_5\gamma_\mu X$

Einstein-Cartan portal to dark matter

The four - fermion interaction opens up the production channel of N-particles through the annihilation of the SM fermions X, via the reaction $X + \bar{X} \to N + \bar{N}$ .The kinetic equation corresponding to this reaction takes the form

$$\left(\frac{\partial}{\partial t} - Hq_i \frac{\partial}{\partial q_i}\right) f_N(t, \vec{q}\,) = R(\vec{q}\,, T)$$

Maximal production temperature

Abundance:

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 3.6 \cdot 10^{-2} C_f \left(\frac{M_N}{10\text{keV}}\right) \left(\frac{T_{\text{prod}}}{M_P}\right)^3$$

with coefficient C_f is different for Dirac and Majorana fermions,

$$C_M = \frac{9}{4}\left\{24\left(1+\alpha^2-\beta^2\right)^2 + 21\left(1-(\alpha+\beta)^2\right)^2\right\}$$

$$C_D = \frac{9}{4}\left\{45\left(1+\alpha^2-\beta^2\right)^2 + 21\left(1-(\alpha+\beta)^2\right)^2 + 24\left(1-(\alpha-\beta)^2\right)^2\right\}$$

Einstein-Cartan portal to dark matter

After the Higgs inflation the reheating is almost instantaneous (DeCross, Kaiser, Prabhu, Prescod-Weinstein, Sfakianakis; Ema, Jinno, Mukaida, Nakayama ; Rubio, Tomberg; Bezrukov, Shepherd), so we can take $T_{\rm prod} = T_{\rm reh}$, with

$$T_{\rm reh} \simeq \left(\frac{15\lambda}{2\pi^2 g_{\rm eff}}\right)^{\frac{1}{4}} \frac{M_P}{\sqrt{\xi}}$$

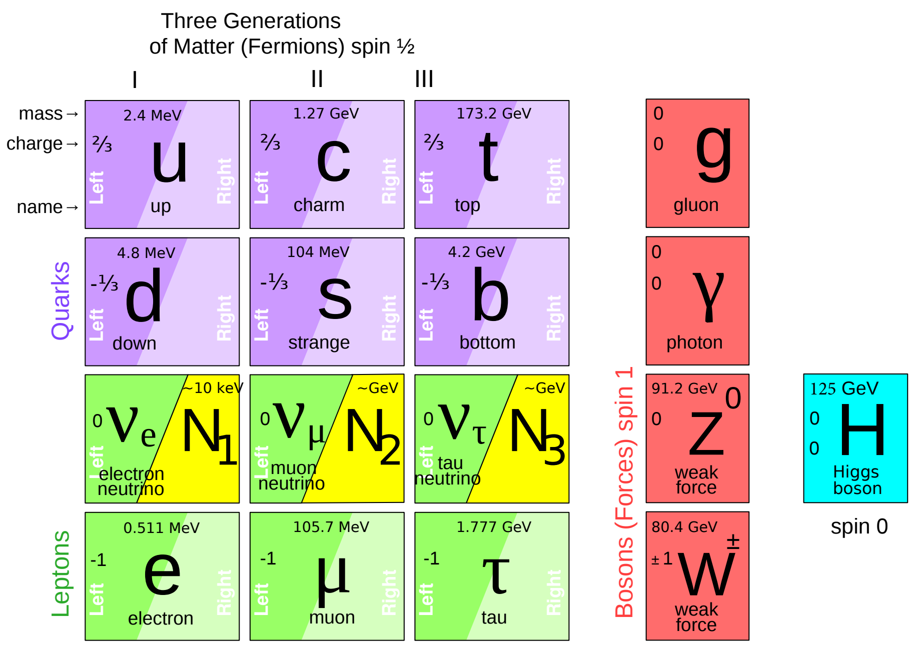
Two "natural" choices of non-minimal couplings α, β:

- $\alpha = \beta = 0$ (absence of non-minimal couplings). Then for Palatini Higgs inflation the correct DM abundance is obtained for $(3-6) \times 10^8$ GeV fermion, Dirac or Majorana.
- $\alpha \sim \beta \sim \sqrt{\xi}$ (the universal UV cutoff $\Lambda \sim M_P/\sqrt{\xi}$). Then for Palatini Higgs inflation the correct DM abundance is obtained for a keV scale fermion.

A new mechanism for production of sterile neutrino Dark matter!

Einstein-Cartan portal to dark matter

Application to the νMSM

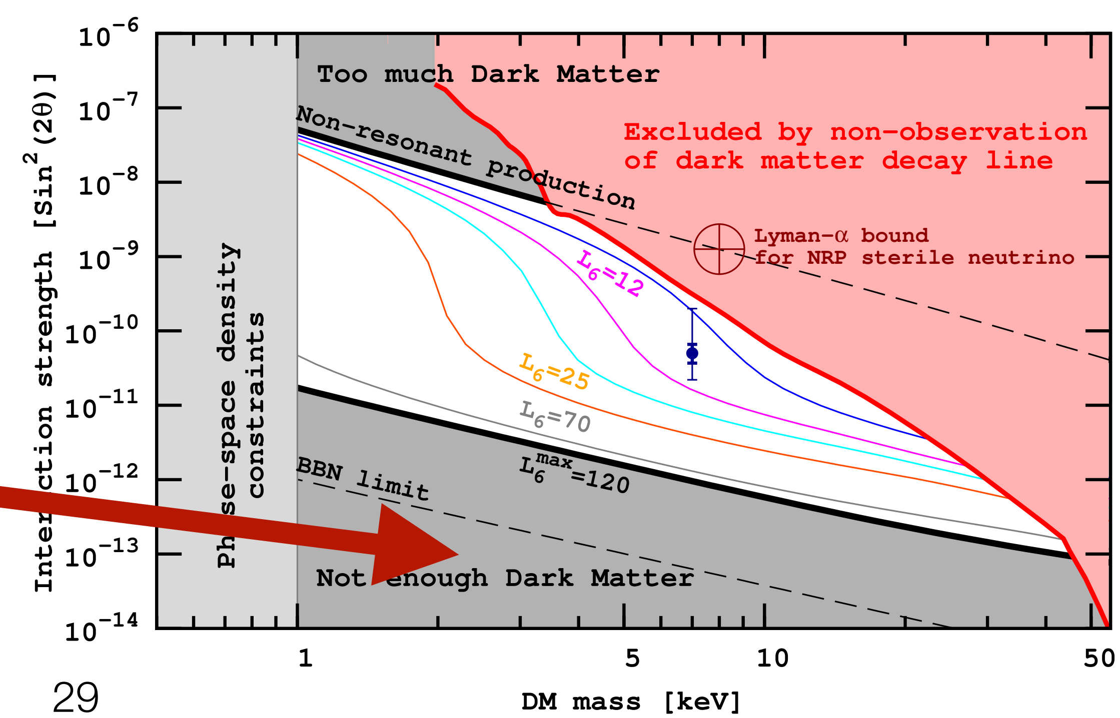


Higgs boson: EW symmetry breaking and inflation

Heavier N_2 and N_3, GeV range - neutrino masses and baryogenesis

Lightest HNL N_1, keV range - dark matter

Lower bound on the sterile neutrino mixing angle disappears!

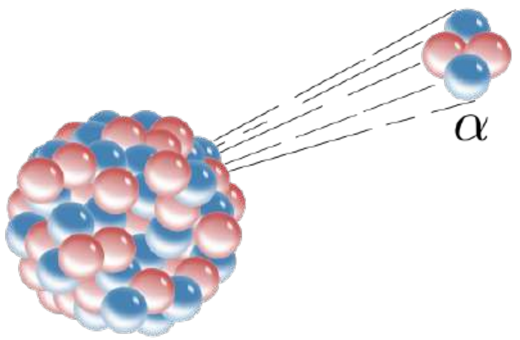


Non-perturbative generation of the electroweak scale

Why the Fermi scale (mass of the Higgs boson, m_H) ~ 100 GeV is so much smaller than the quantum gravity Planck scale M_P ~ 10^{19} GeV? Many proposals: supersymmetry, extra dimensions, and composite Higgs boson, predicting new particles at the Fermi scale, to be found at the LHC. However, none were discovered so far.

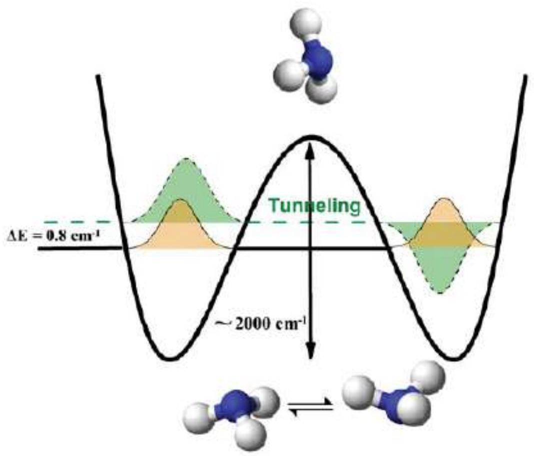
Proposal (Shkerin, MS, 1803.08907): there is only one fundamental scale in Nature - M_P and the electroweak scale is generated from it non-perturbatively. The huge difference between m_H and M_P is due to the tunnelling phenomena in gravity and the Higgs mass is related to the Planck scale as $m_H^2 = M_P^2 e^{-S}$ with S ~ 80.

- 1928, Gamow's theory of α-decay, uranium-238 → thorium-234 + α,



$$\Gamma = E_{\text{bounding}} e^{-S} <<< E_{\text{bounding}}$$

- 1951, Townes, Ammonia Maser,



$$\omega = E_{\text{bounding}} e^{-S} <<< E_{\text{bounding}}$$

- Mass gap in BCS superconductors

$$T_c \sim E_D e^{-1/NV}$$

Fermi scale generation

Scalar theory plus EC gravity. For simplicity let's take the theory without Holst and Nieh-Yan terms (i.e. gravity in Palatini formulation). The action (metric -+++):

$$\frac{\mathscr{L}_{\varphi,g}}{\sqrt{g}} = \frac{1}{2}(M_P^2 + \xi\varphi^2)R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

Scale invariant "matter" with $V(\varphi) = \frac{\lambda}{4}\varphi^4$. The cutoff of the theory (onset of perturbation theory breaking) $\Lambda \sim M_P/\sqrt{\xi}$

We want to compute the Higgs vev:

$$\langle\varphi\rangle \sim \int \mathscr{D}\varphi \mathscr{D}g_{\mu\nu}\varphi e^{-S_E}$$

S_E is the euclidean action of the model.

Remarks:

- Euclidean path integral for gravity may not be well defined due to the problem with the conformal factor of the metric
- We will ignore this problem and follow the crowd: Hawking; Coleman, de Luccia; Veneziano; ..., Isidori, Rychkov, Strumia, Tetradis; ... Branchina, Messina, Sher;..

Fermi scale generation

For small $\varphi \ll M_P$ - gravity is irrelevant – no contribution to the vev of the Higgs from scalar loops.

- Challenge: account for contributions with $\varphi \gg M_P$. Theory for large φ:

$$\mathscr{L} = -\frac{1}{2}\xi\varphi^2 R + \frac{1}{2}(\partial\varphi)^2 + \frac{\lambda}{4}\varphi^4$$

Important properties of this action:

- Scale-invariance
- Planck scale is dynamical, $M_P(\varphi) \propto \sqrt{\xi}\varphi$
- Conjecture: contribution of large Higgs fields $\varphi > M_P/\sqrt{\xi}$ to path integral is better to be found in the Einstein frame
- Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{d\varphi} = \frac{1}{\Omega} \quad \Longrightarrow \quad \varphi = \frac{M_P}{\sqrt{\xi}} \sinh\left(\frac{\sqrt{\xi}\chi}{M_P}\right)$$

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Resulting action

$$S = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} + \frac{\lambda M_P^4}{4\xi^2}\sinh^4\left(\frac{\sqrt{\xi}\chi}{M_P}\right)\right\}$$

source term

Most important:

$$\langle\varphi(x)\rangle \sim \int \mathscr{D}\mathscr{A}\mathscr{D}\varphi(x)\mathscr{D}g_{\mu\nu}\varphi(x)e^{-S_E} \Longrightarrow \int \mathscr{D}\mathscr{A}\mathscr{D}\chi\mathscr{D}\hat{g}_{\mu\nu}e^{\frac{\sqrt{\xi}\chi(x)}{M_P}-S_E}$$

Modification of the action and equations of motion!

Equations of motion for χ contain a source term $\delta(x) \longrightarrow$ new classical solutions.

Semiclassical parameter: $\sqrt{\xi}$, analogue of $1/\hbar$ in WKB approximation.

Similar to

- computation of $\int dx x^N e^{-x^2}$ for large N ,

- computation of multi-particle production in Khlebnikov, Rubakov, Tinyakov '91,

- proof of confinement in 3D Georgi-Glashow model, $\langle\exp(\int A_\mu dv^\mu)\rangle$ in Polyakov '76.

Fermi scale generation

Field equations in the Einstein frame for maximally O(4) symmetric metric

$$ds^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$$

$$\partial_r\left(\frac{r^3\chi'}{f}\right) - r^3 f U'(\chi) = -\frac{\sqrt{\xi}}{2\pi^2 M_P}\delta(r)$$

$$6 - 6f^2 + \frac{2r^2f^2U(\chi)}{M_P^2} - \frac{r^2\chi'^2}{M_P^2} = 0$$

Boundary conditions at infinity : $r \to \infty$: $f^2(r) \to 1, \chi(r) \to 0$.

The action is singular, since $\chi \to \infty$ when r → 0. Add to the action higher dimensional operators, regularising the singularity. We also require that these operators:

- do not introduce new degrees of freedom
- do not spoil asymptotic scale invariance, when h → ∞

Example (other operators work as well) : $\delta\mathscr{L}_\delta = -\frac{\delta}{M_P^8\Omega^8}\left(1+\frac{\delta}{\Omega^2}\right)\left(\partial_\mu h\right)^6$

Fermi scale generation

Natural choice: $\delta \sim \xi^2$: the same cutoff $\Lambda \sim M_P/\sqrt{\xi}$

Requirement of the correct hierarchy. Parametrically $S \propto \sqrt{\xi}$

Constraint from inflation

Constraint from t-quark mass: positive λ at the scale of inflation

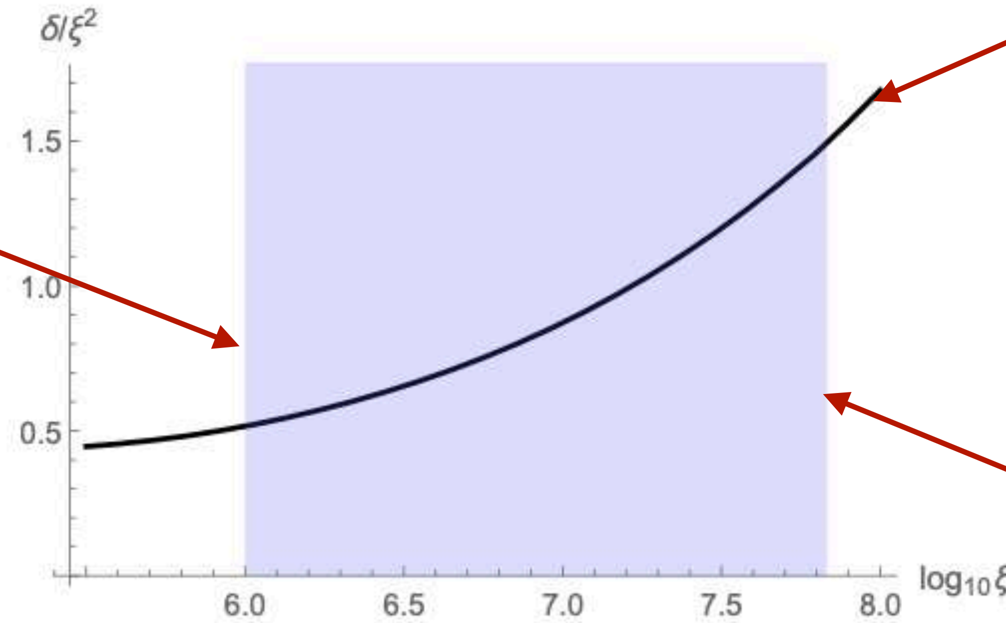

Figure 1. Values of the non-minimal coupling ξ and the coupling δ of the higher-order operator (11), for which $B = \ln(M_P/(\sqrt{\xi}M_F))$. Admissible values of ξ are within the blue area, the left bound coming from inflation and the right bound coming from top quark measurements.

See also:
Rasanen and Rasanen, 1709.07853;
Rasanen, 1811.09514;
Karananas, Michel and Rubio, 2006.11290

The hierarchy between the Planck and the Fermi scales may be a natural phenomenon when the SM is classically conformal, ξ is large and the gravity is of the Palatini type! In the metric theory the source term $\sqrt{\xi}\delta(r)$ is replaced by $\delta(r)/(6+1/\xi)$ and the action is too small.

Conclusions

Einstein-Cartan gravity is an interesting theory with the following properties:

- It has the same number of degrees of freedom as the metric gravity
- Higgs inflation is a natural consequence of EC gravity with universal prediction for $n_s = 1 - 2/N \simeq 0.96$, and with r which can be as small as 10^{-10} and as large as the present experimental limit
- It leads to a new universal mechanism for fermion dark matter production operating for masses as small as few keV and as large as $(3-6) \times 10^8$ GeV
- It may lead to an explanation why the Fermi scale is much smaller than the Planck scale, using non-perturbative semiclassical effects - new type of singular instanton