QCD at non-zero density and phenomenology

CLAUDIA RATTI
UNIVERSITY OF HOUSTON
Matter in the Universe

Two- and three-quark states only!
Matter in the Universe

Quark-Gluon Plasma: new phase of matter at very high temperatures (or densities)
QCD matter under extreme conditions

Research Council of the National Academies: Eleven science questions for the new century

- How did the Universe begin?
- What are the new states of matter at exceedingly high density and temperature?
- What is Dark Matter?
- What is the nature of Dark Energy?
- What are the masses of the neutrinos, how have they shaped the evolution of the Universe?
- Did Enstein have the last word on Gravity?
- How do cosmic accelerators work and what are they accelerating?
- Are protons unstable?
- Are there additional space-time dimensions?
- How were the elements from Iron to Uranium made?
- Is a new theory of matter and light needed at the highest energies?
QCD matter under extreme conditions

Research Council of the National Academies: Eleven science questions for the new century

- How did the Universe begin?
- What are the new states of matter at exceedingly high density and temperature?

The two questions are related!
Quark-Gluon Plasma (QGP) is at $T > 10^{12} \text{K}$ and $\rho \sim 10^{40} \text{ cm}^{-3}$
The Universe was in the QGP phase a few $\mu$s after Big Bang
Ultimate goals

Phase diagram of water

Graphics credit to: ООО ИнтерГрафика
Ultimate goals

Phase diagram of strongly interacting matter

Graphics credit to: ООО ИнтерГрафика
Open Questions

- Is there a critical point in the QCD phase diagram?
- What are the degrees of freedom in the vicinity of the phase transition?
- Where is the transition line at high density?
- What are the phases of QCD at high density?
- Are we creating a thermal medium in experiments?
QCD matter under extreme conditions

To address these questions, we need fundamental theory and experiment.

**Theory: Quantum Chromodynamics**

- QCD is the fundamental theory of strong interactions
- It describes interactions among quarks and gluons

\[ L_{QCD} = \sum_{i=1}^{n_f} \bar{\psi}_i \gamma_\mu \left( i \partial^\mu - g A^\mu_a \frac{\lambda_a}{2} \right) \psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} \sum_a F_{a\mu\nu} F^{a\mu\nu} \]

\[ F_{a\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - i f_{abc} A_b^\mu A_c^\nu \]

- Quark-Gluon Plasma (QGP) discovery at RHIC and LHC:
  - SURPRISE!!! QGP is a PERFECT FLUID

- Changes our idea of QGP
  (no weak coupling)
- Microscopic origin still unknown
Gold nuclei, with 197 protons + neutrons each, are accelerated.
The beams go through the experimental apparatus 100,000 times per second!
Second Beam Energy Scan (BESII) at RHIC

- Planned for 2019-2020
- 24 weeks of runs each year
- Beam Energies have been chosen to keep the $\mu_B$ step $\sim 50$ MeV
- Chemical potentials of interest: $\mu_B/T \sim 1.5...4$

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<th>$\sqrt{s}$ (GeV)</th>
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<th>14.5</th>
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## Comparison of the facilities

<table>
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<tr>
<th>Facility</th>
<th>RHIC BESII</th>
<th>SPS</th>
<th>NICA</th>
<th>SIS-100</th>
<th>J-PARC HI</th>
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<tr>
<td>Exp.:</td>
<td>STAR + FXT</td>
<td>NA61</td>
<td>MPD + BM@N</td>
<td>SIS-300 CBM</td>
<td>JHITS</td>
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<td>4.9-17.3</td>
<td>2.7-11</td>
<td>2.7-8.2</td>
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<td>2.0-3.5</td>
<td>&lt;10 MHz</td>
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<td>Fixed target Lighter ion collisions</td>
<td>Collider Fixed target</td>
<td>Fixed target</td>
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</tr>
</tbody>
</table>

**Compilation by D. Cebra**

CP=Critical Point  OD= Onset of Deconfinement  DHM=Dense Hadronic Matter
The theory of strong interactions

- Quantum ChromoDynamics (QCD) Nobel prize 2004
- Analytic solutions of QCD are not possible in the non-perturbative regime
- Numerical approach to solve QCD
- Simulations are running on the most powerful supercomputers in the world

Fundamental fields

Gauge fields: $U_\mu(x)$, SU(3) live on the links ($\mu$ index)

Quark fields: $\Psi(x)$, $\bar{\Psi}(x)$, anti-commuting Grassmann variables live on the sites

Wilson fermions: computationally expensive

Staggered fermions: faster, but taste symmetry violation (only one pseudogoldstone pion instead of three)

Fermion doubling is avoided by rooting: "good, bad or ugly?"
How can lattice QCD support the experiments?

- **Equation of state**
  - Needed for *hydrodynamic* description of the QGP

- **QCD phase diagram**
  - Transition line at finite density
  - Constraints on the location of the critical point

- **Fluctuations of conserved charges**
  - Can be *simulated* on the lattice and *measured* in experiments
  - Can give information on the *evolution* of heavy-ion collisions
  - Can give information on the *critical point*
QCD Equation of State at finite density

TAYLOR EXPANSION

ANALYTICAL CONTINUATION FROM IMAGINARY CHEMICAL POTENTIAL

ALTERNATIVE EQUATION OF STATE AT LARGE DENSITIES
QCD EoS at $\mu_B=0$

- EoS for $N_f=2+1$ known in the continuum limit since 2013
- Good agreement with the HRG model at low temperature
- Charm quark relevant degree of freedom already at $T\sim 250$ MeV
Constraints on the EoS from the experiments

- Comparison of data from RHIC and LHC to theoretical models through Bayesian analysis
- The posterior distribution of EoS is consistent with the lattice QCD one
Taylor expansion of EoS

- Taylor expansion of the pressure:

\[
\frac{p(T, \mu_B)}{T^4} = \frac{p(T, 0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{d^{2n}(p/T^4)}{d(\mu_B/T)^{2n}} \bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}
\]

- Two ways of extracting the Taylor expansion coefficients:
  - Direct simulation
  - Simulations at imaginary \( \mu_B \)

- Two physics choices:
  - \( \mu_B \neq 0, \mu_S = \mu_Q = 0 \)
  - \( \mu_S \) and \( \mu_Q \) are functions of \( T \) and \( \mu_B \) to match the experimental constraints:
    \[
    <n_S> = 0 \quad \quad <n_Q> = 0.4 <n_B>
    \]
Simulations at imaginary $\mu_B$:
Continuum, $O(10^4)$ configurations, errors include systematics (WB: NPA (2017))

**Strangeness neutrality**

New results for $\chi_n^B = n!c_n$ at $\mu_S = \mu_Q = 0$ and $N_t = 12$
We now have the equation of state for $\mu_B/T \leq 2$ or in terms of the RHIC energy scan:

$$\sqrt{s} = 200, \ 62.4, \ 39, \ 27, \ 19.6, \ 14.5 \text{GeV}$$
**Alternative EoS at large densities**

P. Parotto, C. R. et al., PRC (2020)

- EoS for QCD with a 3D-Ising critical point

\[ T_{4c_n}^{\text{LAT}}(T) = T_{4c_n}^{\text{Non-Ising}}(T) + T_c^{4c_n} \text{Ising}(T) \]

- Implement scaling behavior of 3D-Ising model EoS
- Define map from 3D-Ising model to QCD
- Estimate contribution to Taylor coefficients from 3D-Ising model critical point
- Reconstruct full pressure

**Entropy and baryon density discontinuous at \( \mu_B > \mu_{Bc} \)**

QCD phase diagram

- Transition Temperature
- Transition Line
- Transition Width
Phase Diagram from Lattice QCD

- The transition at $\mu_B=0$ is a smooth crossover

Borsanyi et al., JHEP (2010)
Bazavov et al., PRD (2012)
QCD transition temperature and curvature

\[ \frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left( \frac{\mu_B}{T_0} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T_0} \right)^4 + \mathcal{O}(\mu_B^6) \]

- QCD transition at \( \mu_B = 0 \) is a crossover

- Latest results on \( T_0 \) from WB collaboration based on subtracted chiral condensate and chiral susceptibility

\[ T_0 = 158.0 \pm 0.6 \text{ MeV} \]

Compilation by F. Negro

\[ \kappa_2 = 0.0153 \pm 0.0018, \quad \kappa_4 = 0.00032 \pm 0.00067 \]
For a genuine phase transition, the height of the peak of the chiral susceptibility diverges and the width shrinks to zero.

No sign of criticality for $\mu_B < 300$ MeV.

Borsanyi, C. R. et al. PRL (2020)
Fluctuations of conserved charges

Comparison to experiment: chemical freeze-out parameters

Off-diagonal correlators
Evolution of a heavy-ion collision

- **Chemical freeze-out**: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- **Kinetic freeze-out**: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- **Hadrons reach the detector**
Freeze-out vs phase transition
Distribution of conserved charges

- Consider the number of electrically charged particles $N_Q$
- Its average value over the whole ensemble of events is $<N_Q>$
- In experiments it is possible to measure its event-by-event distribution

STAR Collab.: PRL (2014)
Cumulants of multiplicity distribution

Deviation of $N_Q$ from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$

The cumulants of the event-by-event distribution of $N_Q$ are:

- $\chi_2 = \langle (\delta N_Q)^2 \rangle$
- $\chi_3 = \langle (\delta N_Q)^3 \rangle$
- $\chi_4 = \langle (\delta N_Q)^4 \rangle - 3 \langle (\delta N_Q)^2 \rangle^2$

The cumulants are related to the central moments of the distribution by:

- Variance: $\sigma^2 = \chi_2$
- Skewness: $S = \chi_3 / (\chi_2)^{3/2}$
- Kurtosis: $\kappa = \chi_4 / (\chi_2)^2$

![Graph of cumulants and central moments](image)
Fluctuations on the lattice

- **Fluctuations** of conserved charges are the **cumulants** of their event-by-event distribution.

- Definition:
  \[
  \chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}.
  \]

- They can be calculated on the lattice and compared to experiment.

- Variance: \( \sigma^2 = \chi_2 \)
  
  Skewness: \( S = \chi_3 / (\chi_2)^{3/2} \)

  Kurtosis: \( \kappa = \chi_4 / (\chi_2)^2 \)

\[
\begin{align*}
  S\sigma &= \chi_3 / \chi_2 \\
  \kappa\sigma^2 &= \chi_4 / \chi_2 \\
  M/\sigma^2 &= \chi_1 / \chi_2 \\
  S\sigma^3 / M &= \chi_3 / \chi_1
\end{align*}
\]
Freeze-out line from first principles

- Use $T$- and $\mu_B$-dependence of $R_{12}^Q$ and $R_{12}^B$ for a combined fit:

$$R_{12}^Q(T, \mu_B) = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} = \frac{\chi_{11}^{QB}(T, 0) + \chi_2^Q(T, 0)q_1(T) + \chi_{11}^{QS}(T, 0)s_1(T)}{\chi_2^Q(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

C. Ratti for WB, NPA (2017)
What about strangeness?

- Data for net-kaon fluctuations seem to prefer a higher freeze-out temperature.
  

- Separate analysis of particle yields gives a similar result


F. Flor et al., 2009.14781 (2020)
Off-diagonal fluctuations of conserved charges

• The measurable species in HIC are only a handful. How much do they tell us about the correlation between conserved charges?

• Historically, the proxies for B, Q and S have been p, p, π, K and K themselves → what about off-diagonal correlators?

• We want to find:
  • The main contributions to off-diagonal correlators
  • A way to compare lattice to experiment

Off-diagonal correlators

- The species that are stable **under strong interactions**, AND are **measurable**
  \[ \pi^\pm, K^\pm, p\bar{p}, \Lambda\bar{\Lambda}, \Xi^- (\Xi^+) , \Omega^- (\Omega^+) \]
  
  \[ \rightarrow \] we inevitably lose a good chunk of conserved charges!

- Thanks to the **separation between observable and non-observables species**, one can pinpoint what can be measured and what cannot of \( \chi_{ijk}^{BQS} \) R. Bellwied, C. R. et al., PRD (2020)

- For the **proton- and kaon-dominated \( \chi_{BQ} \) and \( \chi_{QS} \)**, a large part of the full correlator is carried by measurable particles
- \( \chi_{BS} \) is less transparent, and requires careful analysis of its contributions
Constructing a proxy not a trivial task: consider main contributions to numerator and denominator

- Good proxy for $\chi_{11}^{BS}/\chi_2^S$: $\tilde{C}_{BS,SS}^{\Lambda,\Lambda} = \sigma_\Lambda^2/(\sigma_K^2 + \sigma_\Lambda^2)$
Fluctuations at the critical point

M. Stephanov, PRL (2009).

1. \( \mu < \mu_{CP} \)

The probability distribution for the order parameter

\[
P[\sigma] \sim \exp \left\{ -\frac{\Omega[\sigma]}{T} \right\}
\]

\[
\Omega = \int d^3x \left[ \frac{1}{2} (\nabla \sigma)^2 + \frac{m_\sigma^2}{2} \sigma_2 + \frac{\lambda_3}{3} \sigma^3 + \ldots \right]
\]

2. \( \mu = \mu_{CP} \)

The correlation length (\( \xi = m_\sigma^{-1} \))

\[
\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0
\]

\[
\chi_2 = VT \xi^2
\]

\[
\chi_3 = 2VT^{3/2}\hat{\lambda}_3 \xi^{9/2}
\]

\[
\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4] \xi^7
\]

3. \( \mu > \mu_{CP} \)
A different approach at large densities

- Use AdS/CFT correspondence
- Fix the parameters to reproduce everything we know from the lattice
- Calculate observables at finite density
- Fluctuations of conserved charges: they are sensitive to the critical point
Black Hole Susceptibilities

The black hole model contains a critical end point at

\[ \mu_B = 723 \pm 36 \text{ MeV} \quad \text{and} \quad T = 89 \pm 11 \text{ MeV} \]
Conclusions

- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 2$
- Extensions to higher densities by means of lattice-based models
- No indication of Critical Point from lattice QCD in the explored $\mu_B$ range
Backup slides
Hadron Resonance Gas model

Dashen, Ma, Bernstein; Prakash, Venugopalan; Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

\[ \frac{p^{\text{HRG}}}{T^4} = \frac{1}{VT^3} \sum_{i \in \text{mesons}} \ln Z^M_{m_i}(T, V, \mu X^a) + \frac{1}{VT^3} \sum_{i \in \text{baryons}} \ln Z^B_{m_i}(T, V, \mu X^a) \]

where

\[ \ln Z^M_{m_i}/B = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) , \]

with energies \( \varepsilon_i = \sqrt{k^2 + m_i^2} \), degeneracy factors \( d_i \) and fugacities

\[ z_i = \exp \left( \sum_a X^a_i \mu X^a / T \right) . \]

**\( X^a \)**: all possible conserved charges, including the baryon number \( B \), electric charge \( Q \), strangeness \( S \).

- Fugacity expansion for \( \mu_s=\mu_Q=0 \):

\[ \frac{p_B}{T^4} = \sum_{i \in B} \frac{d_i}{\pi^2} \left( \frac{m_i}{T} \right)^2 \sum_{N=1}^\infty (-1)^{N+1} N^{-2} K_2(N m_i / T) \cosh \left[ N \frac{\mu_B}{T} \right] \]

Boltzmann approximation: \( N=1 \)
Kaon fluctuations on the lattice

Lattice QCD works in terms of conserved charges

Challenge: isolate the fluctuations of a given particle species

Assuming an HRG model in the Boltzmann approximation, it is possible to write the pressure as:

\[
P(\hat{\mu}_B, \hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(\hat{\mu}_S) \\
+ P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)
\]

Kaons in heavy ion collisions: primordial + decays

Idea: calculate $\chi_2^K/\chi_1^K$ in the HRG model for the two cases: only primordial kaons in the Boltzmann approximation vs primordial + resonance decay kaons
Kaon fluctuations on the lattice

- Boltzmann approximation works well for lower order kaon fluctuations
  \[
  \frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}
  \]
- \(\chi_2^K/\chi_1^K\) from primordial kaons + decays is very close to the Boltzmann approximation
- \(\mu_S\) and \(\mu_Q\) are functions of \(T\) and \(\mu_B\) to match the experimental constraints:

J. Noronha-Hostler, C.R. et al., forthcoming
Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution
  - A. Bzdak, V. Koch, PRC (2012)

- Spallation protons
  - Experimentally removed with proper cuts in $p_T$

- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance

- Baryon number conservation
  - Experimental data need to be corrected for this effect
  - P. Braun-Munzinger et al., NPA (2017)

- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations
  - M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test
  - J. Steinheimer et al., PRL (2013)
Fluctuations at the critical point

M. Stephanov, PRL (2009).

- Correlation length near the critical point
  \[ \xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0 \]
  \[ \chi_2 = VT\xi^2 \]
  \[ \chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2} \]
  \[ \chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7 \]

- Fluctuations are expected to diverge at the critical point
- Fourth-order fluctuations should have a non-monotonic behavior
- Preliminary STAR data seem to confirm this
- Can we describe this trend with lattice QCD?
Fluctuations along the QCD crossover

Net-baryon variance

\[
\frac{\sigma_{B}^2(T_c(\mu_B),\mu_B) - \sigma_{B}^2(T_0,0)}{\sigma_{B}^2(T_0,0)} = \lambda_2 \left(\frac{\mu_B}{T_0}\right)^2 + \lambda_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)
\]

\[
\frac{\sigma_{B_{1}}^2(T_c(\mu_B),\mu_B)}{\sigma_{B_{1}}^2(T_0,0)} - 1
\]

- \(\mathcal{O}(\mu_B^4)\)
- \(n_s = 0\), \(\frac{n_s}{n_B} = 0.4\)
- HRG

Disconnected chiral susceptibility

\[
\chi_{\text{sub}} \equiv \frac{T}{V} m_s \left[ \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right] \left[ m_s (\Sigma_u + \Sigma_d) - (m_u + m_d) \Sigma_s \right]
\]

- Expected to be larger than HRG model result near the CP
- No sign of criticality

- Peak height expected to increase near the CP
- No sign of criticality

See talk by Patrick Steinbrecher on Wednesday
Higher order fluctuations

HotQCD, PRD (2017)

Alternative explanation: canonical suppression

A. Rustamov
@QM2018
Off-diagonal correlators

- Simulation of the lower order correlators at imaginary $\mu_B$
- Fit to extract higher order terms
- Results exist also for BS, QS and BQS correlators

\[
\begin{align*}
\chi^{BS}_{11}(\hat{\mu}_B) &= \chi^{BS}_{11} + \frac{1}{2!} \chi^{BS}_{31} \hat{\mu}_B^2 + \frac{1}{4!} \chi^{BS}_{51} \hat{\mu}_B^4 + \frac{1}{6!} \chi^{BS}_{71} \hat{\mu}_B^6 + \frac{1}{8!} \chi^{BS}_{91} \hat{\mu}_B^8 \\
\chi^{BS}_{21}(\hat{\mu}_B) &= \chi^{BS}_{31} \hat{\mu}_B + \frac{1}{3!} \chi^{BS}_{51} \hat{\mu}_B^3 + \frac{1}{5!} \chi^{BS}_{71} \hat{\mu}_B^5 + \frac{1}{7!} \chi^{BS}_{91} \hat{\mu}_B^7 \\
\chi^{BS}_{31}(\hat{\mu}_B) &= \chi^{BS}_{31} + \frac{1}{2!} \chi^{BS}_{51} \hat{\mu}_B^2 + \frac{1}{4!} \chi^{BS}_{71} \hat{\mu}_B^4 + \frac{1}{6!} \chi^{BS}_{91} \hat{\mu}_B^6
\end{align*}
\]

Forthcoming experimental data at RHIC
See talk by Jana Guenther on Wednesday
• Simulation of the lower order correlators at imaginary $\mu_B$
• Fit to extract higher order terms
• Results exist also for BS, QS and BQS correlators

$$\chi^{BS}_{11} (\hat{\mu}_B) = \chi^{BS}_{11} + \frac{1}{2!} \chi^{BS}_{31} \hat{\mu}_B^2 + \frac{1}{4!} \chi^{BS}_{51} \hat{\mu}_B^4 + \frac{1}{6!} \chi^{BS}_{71} \hat{\mu}_B^6 + \frac{1}{8!} \chi^{BS}_{91} \hat{\mu}_B^8$$

$$\chi^{BS}_{21} (\hat{\mu}_B) = \chi^{BS}_{31} \hat{\mu}_B + \frac{1}{3!} \chi^{BS}_{51} \hat{\mu}_B^3 + \frac{1}{5!} \chi^{BS}_{71} \hat{\mu}_B^5 + \frac{1}{7!} \chi^{BS}_{91} \hat{\mu}_B^7$$

$$\chi^{BS}_{31} (\hat{\mu}_B) = \chi^{BS}_{31} + \frac{1}{2!} \chi^{BS}_{51} \hat{\mu}_B^2 + \frac{1}{4!} \chi^{BS}_{71} \hat{\mu}_B^4 + \frac{1}{6!} \chi^{BS}_{91} \hat{\mu}_B^6$$

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- Simulation of the lower order correlators at imaginary $\mu_B$
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\[
\begin{align*}
\chi_{11}^{BS}(\hat{\mu}_B) &= \chi_{11}^{BS} + \frac{1}{2!} \chi_{31}^{BS} \hat{\mu}_B^2 + \frac{1}{4!} \chi_{51}^{BS} \hat{\mu}_B^4 + \frac{1}{6!} \chi_{71}^{BS} \hat{\mu}_B^6 + \frac{1}{8!} \chi_{91}^{BS} \hat{\mu}_B^8 \\
\chi_{21}^{BS}(\hat{\mu}_B) &= \chi_{31}^{BS} \hat{\mu}_B + \frac{1}{3!} \chi_{51}^{BS} \hat{\mu}_B^3 + \frac{1}{5!} \chi_{71}^{BS} \hat{\mu}_B^5 + \frac{1}{7!} \chi_{91}^{BS} \hat{\mu}_B^7 \\
\chi_{31}^{BS}(\hat{\mu}_B) &= \chi_{31}^{BS} + \frac{1}{2!} \chi_{51}^{BS} \hat{\mu}_B^2 + \frac{1}{4!} \chi_{71}^{BS} \hat{\mu}_B^4 + \frac{1}{6!} \chi_{91}^{BS} \hat{\mu}_B^6
\end{align*}
\]
Other approaches I did not have time to address

- Reweighting techniques (Fodor & Katz)
- Canonical ensemble (Alexandru et al., Kratochvila, de Forcrand, Ejiri, Bornyakov, Goy, Lombardo, Nakamura)
- Density of state methods (Fodor, Katz & Schmidt, Alexandru et al.)
- Two-color QCD (ITEP Moscow lattice group, Kogut et al., S. Hands et al., von Smekal et al.)
- Scalar field theories with complex actions (See talk by M. Ogilvie on Tuesday)
- Complex Langevin (see talks by D. Sinclair, S. Tsutsui, F. Attanasio, Y. Ito, A. Joseph on Monday)
- Lefshetz Thimble (see talks by K. Zambello, S. Lawrence, N. Warrington, H. Lamm on Monday)
- Phase unwrapping (see talks by G. Kanwar and M. Wagman on Friday)
Conclusions

- Need for quantitative results at finite-density to support the experimental programs
  - Equation of state
  - Phase transition line
  - Fluctuations of conserved charges
- Current lattice results for thermodynamics up to $\mu_B/T \leq 2$
- Extensions to higher densities by means of lattice-based models
- No indication of Critical Point from lattice QCD in the explored $\mu_B$ range
Lattice QCD temperatures have a large uncertainty but they are above the light flavor ones.
Fluctuations of conserved charges?

🌟 If we look at the entire system, none of the conserved charges will fluctuate

🌟 By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful

- $\Delta Y_{\text{total}}$: range for total charge multiplicity distribution
- $\Delta Y_{\text{accept}}$: interval for the accepted charged particles
- $\Delta Y_{\text{kick}}$: rapidity shift that charges receive during and after hadronization
QCD matter under extreme conditions

To address these questions we need fundamental theory and experiment

Theory: Quantum Chromodynamics
• QCD is the fundamental theory of strong interactions
• It describes interactions among quarks and gluons

\[ L_{\text{QCD}} = \sum_i \bar{\psi}_i \gamma_\mu \left( i \partial^\mu - g A_\mu^a \frac{\lambda_a}{2} \right) \psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} \sum_a F_{\mu\nu}^a F^{\mu\nu}_a \]

Confinement

Asymptotic Freedom

Experiment: heavy-ion collisions
• Quark-gluon plasma (QGP) discovery at RHIC and the LHC
• QGP is a strongly interacting (almost) perfect fluid
Cumulants of multiplicity distribution

- Deviation of $N_Q$ from its mean in a single event: $\delta N_Q = N_Q - <N_Q>$

- The cumulants of the event-by-event distribution of $N_Q$ are:
  
  $\chi_2 = <(\delta N_Q)^2>$  \hspace{1cm} $\chi_3 = <(\delta N_Q)^3>$  \hspace{1cm} $\chi_4 = <(\delta N_Q)^4> - 3<\delta N_Q^2>^2$

- The cumulants are related to the central moments of the distribution by:

  Variance: $\sigma^2 = \chi_2$
  
  Skewness: $S = \chi_3 / (\chi_2)^{3/2}$
  
  Kurtosis: $\kappa = \chi_4 / (\chi_2)^2$
Fluctuations and hadrochemistry

\[ \chi_1^B(T, \mu_B) = \frac{\rho_B(T; \mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T) \sinh(k \mu_B/T) \]

- Consistent with HRG at low temperatures
- Consistent with approach to ideal gas limit
- \( b_2 \) departs from zero at \( T \sim 160 \text{ MeV} \)
- Deviation from ideal HRG

V. Vovchenko et al., PLB (2017)

\[ P(\hat{\mu}_B, \hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(\hat{\mu}_S) + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S) \]

P. Alba et al., PRD (2017)

- Need of additional strange hadrons, predicted by the Quark Model but not yet detected
- First pointed out in Bazavov et al., PRL(2014)

(see talk by J. Glesaaen on Friday)
Canonical suppression

A. Rustamov @QM2018

above 11.5 GeV CE suppression accounts for measured deviations from GCE
Analytical continuation – illustration of systematics
Analytical continuation – illustration of systematics

Condition: $\chi_8 \lesssim \chi_4 \rightarrow f(\hat{\mu}_B) = a + b\hat{\mu}_B^2 + c\hat{\mu}_B^4 + \frac{b\epsilon}{840}\hat{\mu}_B^6$

Analytical continuation on $N_t = 12$ raw data

Graph showing $\frac{T}{\mu_B} = \frac{d\rho(T)}{d\mu_B}$ for $T = 170\text{MeV}$ and $T = 145\text{MeV}$, with different values of $\epsilon$. The graph plots $(\mu_B/T)^2 = -\hat{\mu}^2$.
Consistency between freeze-out of B and Q

- Independent fit of $R_{12}^Q$ and $R_{12}^B$: consistency between freeze-out chemical potentials.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>$\mu_B^f$ [MeV] (from B)</th>
<th>$\mu_B^f$ [MeV] (from Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>25.8±2.7</td>
<td>22.8±2.6</td>
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<tr>
<td>62.4</td>
<td>69.7±6.4</td>
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<td>39</td>
<td>105±11</td>
<td>101±10</td>
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<tr>
<td>27</td>
<td>-</td>
<td>136±13.8</td>
</tr>
</tbody>
</table>

WB: PRL (2014)
STAR collaboration, PRL (2014)
Geneva with the Large Hadron Collider

Speed: 0.999995 x speed of light
26.2 km circle