Deconstructing Relativistic Fluid Dynamics

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Fluid Dynamics Is Everywhere
The Ubiquitousness of Fluid Dynamics

Based on conservation laws + large separation of length scales

Separation of scales $\rightarrow$ macroscopic: $L$   microscopic: $\ell$

Knudsen number $\quad K_N \sim \frac{\ell}{L} \ll 1$ $\rightarrow$ FLUID
How does one describe fluid dynamics?

Conservation of mass + Newton's 2nd law + isotropy + incompressibility \((\rho_0)\)

\[
\partial_t \vec{V} + (\vec{V} \cdot \vec{\nabla}) \vec{V} + \frac{\vec{\nabla} \cdot \vec{P}}{\rho_0} = \frac{\eta}{\rho_0} \nabla^2 \vec{V} + \mathcal{O}(K_N^2)
\]

Ideal fluid \(\sim \mathcal{O}(K_n^0)\)

\(\sim \mathcal{O}(K_n)\) Higher order

\(\eta \rightarrow\) shear viscosity
Navier-Stokes equations

Valid when $K_N \ll 1$

\[
\partial_t \vec{V} + (\vec{V} \cdot \vec{\nabla}) \vec{V} + \frac{\vec{\nabla} P}{\rho_0} = \frac{\eta}{\rho_0} \nabla^2 \vec{V}
\]

- Notoriously hard nonlinear problem to solve in three dimensions.

- Turbulence

Millennium Prize Problem

“Global existence and smoothness of solutions of the Navier-Stokes equations”

da Vinci, 1508-1513
Frontiers of Fluid Dynamics Behavior “in the lab”

Redefine “macro” scales $\rightarrow$ Nuclear/Particle Physics

Heavy-Ion Collisions

$10^{-10} \text{ m}$

$f_{m} = 10^{-15} \text{ m}$

Natural units: $\hbar = c = k_B = 1$
Frontiers of Fluid Dynamics Behavior “in the sky”

Fluid dynamics in strong gravitational fields

Neutron Star Mergers

Figure by Most et al., PRL (2019)
In this talk we will consider both frontiers.

Let us start with the frontier defined at very small distances ...
Quantum Chromodynamics (QCD)
Quantum Chromodynamics (QCD)

The fundamental theory of the strong interactions

Non-Abelian gauge theory

Asymptotic freedom

QCD coupling

The QCD vacuum

Color confinement

Strong coupling phenomenon !!!

Quarks and gluons can never be truly free

from D. B. Leinweber

X is any quark.

gluon self-interactions

Hadron (pion)
Quark-Gluon Plasma (QGP) in equilibrium

QCD phase transition in the early universe was a crossover
Out-of-equilibrium properties of QCD?

In order to study the question of the QCD “vacuum”, we must turn to a different direction, we should investigate some “bulk” phenomena by distributing large energy over a large volume.

T. D. Lee, Rev. Mod. Phys. 47 (1975)
Heavy Ion Collisions in a Nutshell – The Little Bang

The way to study non-equilibrium hot QCD phenomena in the lab

QGP = The hottest, densest, smallest, most perfect liquid
Nearly perfect fluidity: An emergent property of QCD

QGP behaves as a strongly coupled liquid !!!!

Shear viscosity to entropy density ratio

$$\frac{\eta}{s} \sim 0.05 - 0.2$$

Noronha-Hostler, Betz, JN, Gyulassy, PRL 2016
(Nearly) Perfect fluidity: an emergent property of QCD

QGP behaves as a strongly coupled relativistic fluid !!!

Fig. from Bernhard, Moreland, Bass, Nature Phys., 2019
The unreasonable effectiveness of hydrodynamics in heavy-ion collisions
How does one describe fluid dynamics in relativity?

Conservation laws (energy and momentum)

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \begin{pmatrix}
T_{00} & T_{01} & T_{02} & T_{03} \\
T_{10} & T_{11} & T_{12} & T_{13} \\
T_{20} & T_{21} & T_{22} & T_{23} \\
T_{30} & T_{31} & T_{32} & T_{33}
\end{pmatrix}$$

Energy-Momentum Tensor

$$T^{\mu\nu} = (\varepsilon + P(\varepsilon))u^\mu u^\nu + P(\varepsilon)g^{\mu\nu} + \Pi^{\mu\nu}$$

What about heavy-ion collisions?
At first (< 2010), it seemed that hydrodynamics was justifiable

Very smooth fluid over nuclear length scales

\[ \frac{\partial \epsilon}{\epsilon_0} \sim \frac{1}{L} \]

\[ \ell \sim \frac{1}{T} \sim \frac{1}{\Lambda_{QCD}} \]

**QGP**

Near equilibrium dynamics

Knudsen number

\[ K_N \sim \ell \partial \epsilon < 0.1 \]

Fluid dynamics at scales of the size of a large nucleus
Reality is much more complicated ...

QGP energy density

- Unavoidable quantum fluctuations
- Large spatial gradients at early times

PARADOX: Knudsen number is large but “hydro” still works

This issue must be understood …
What is the smallest droplet of QCD liquid?

Collective behavior at scales of the size of the proton!

How does liquid behavior emerge from QCD?

What goes into relativistic viscous hydro simulations of heavy-ion collisions?

Figure from B. Schenke
Every single viscous relativistic hydrodynamics simulation in heavy-ions you have ever seen was based on ideas due to Israel and Stewart.

Israel and Stewart

"Hydro" in HIC is not simple textbook hydro

Israel-Stewart (IS) theory


Energy-momentum tensor

\[ T_{\mu \nu} \]

\( \rightarrow \) \( \varepsilon, u_\mu, \pi_{\mu \nu}, \Pi \) as dynamical variables

A theory for hydrodynamic fields and non-hydrodynamic fields

Dynamics: \( \nabla_\mu T^{\mu \nu} = 0 \) (energy-momentum conservation)

\[ u^\lambda \nabla_\lambda \Pi + F(\varepsilon, \nabla_\alpha u_\beta, \pi^{\alpha \beta}, \Pi) = 0 \] (bulk) Many terms! Many coefficients!

\[ u^\lambda \nabla_\lambda \pi^{\mu \nu} + F^{\mu \nu}(\varepsilon, \nabla_\alpha u_\beta, \pi^{\alpha \beta}, \Pi) = 0 \] (shear) 14 EOM

+ equation for diffusion
“Israel-Stewart eqs. make sense even far from equilibrium”

Implicit assumption made in current hydro simulations

Decreasing the system size: from AA to pA and pp

Increasing the theoretical uncertainty

- When does this stop behaving hydrodynamically?
- Are there fundamental constraints that must be fulfilled?
“Israel-Stewart eqs. were proven to be causal, under any circumstances, by Israel and Stewart (1979) themselves”
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False

This rating indicates that the primary elements of a claim are demonstrably false.
“Israel-Stewart eqs. were proven to be causal, under any circumstances, by Israel and Stewart themselves”

False

- Israel and Stewart (1979) proved causality only in the linearized regime around equilibrium.
  - True

- Such results say NOTHINg about the theory in the far from equilibrium regime probed in heavy-ions.

\[
\frac{\pi_{\mu\nu}}{\varepsilon + P}, \quad \frac{\Pi}{\varepsilon + P} \sim \mathcal{O}(1)
\]

- Known to occur in the initial state and at the edges of QGP

Relativistic fluids far from equilibrium: Constraints


Causality in the \textit{nonlinear regime}: shear + bulk effects

\[
\tau_\Pi u^\mu \nabla_\mu \Pi + \Pi = -\zeta \nabla_\mu u^\mu - \delta_{\Pi\Pi} \Pi \nabla_\mu u^\mu - \lambda_{\Pi\Pi} \Pi \mu \nu \sigma_{\mu\nu},
\]

\[
\tau_\Pi \Delta^{\mu\nu}_{\alpha\beta} u^\lambda \nabla_\lambda \Pi^{\alpha\beta} + \Pi^{\mu\nu} = -2\eta \sigma^{\mu\nu} - \delta_{\Pi\Pi} \Pi \mu \nu \nabla_\alpha u^\alpha - \tau_\Pi \tau_\Pi \Pi^{\mu\nu} \sigma^{\mu\nu} - \lambda_{\Pi\Pi} \Pi \sigma^{\mu\nu},
\]

Necessary constraints for meaningful evolution

(2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{1}{2} \tau_{\Pi\Pi} \Pi \geq 0

\[\varepsilon + \Pi + \Pi - \frac{1}{2} \tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{\tau_{\Pi\Pi}}{4} \Pi \geq 0,\]

\[\varepsilon + \Pi + \Pi - \frac{1}{2} \tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) + \lambda_{\Pi\Pi} \Pi \Delta_\Pi + \frac{1}{2} \lambda_{\Pi\Pi} \Pi \Delta_\Pi + \zeta + \delta_{\Pi\Pi} \Pi \lambda_{\Pi\Pi} \Pi \Delta_\Pi + (\varepsilon + \Pi + \Pi + \frac{c^2}{\tau_{\Pi\Pi}}) \geq 0,\]

\[\varepsilon + \Pi + \Pi - \frac{1}{2} \tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{\tau_{\Pi\Pi}}{6} \Pi + (2\eta + \lambda_{\Pi\Pi} \Pi) + (6\delta_{\Pi\Pi} - \tau_{\Pi\Pi}) \Pi \Delta_\Pi + \lambda_{\Pi\Pi} \Pi \Delta_\Pi + (\varepsilon + \Pi + \Pi + \frac{c^2}{\tau_{\Pi\Pi}}) \geq 0,\]

\[\left[\tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{\tau_{\Pi\Pi}}{4} \Pi \right] \geq 0,\]

\[\varepsilon + \Pi + \Pi - (\lambda_{\Pi\Pi} \Pi) - \frac{1}{2} \tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{\tau_{\Pi\Pi}}{4} \Pi \geq 0,\]

\[\varepsilon + \Pi + \Pi - (\lambda_{\Pi\Pi} \Pi) - \frac{1}{2} \tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{\tau_{\Pi\Pi}}{4} \Pi \geq 0,\]

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\[\varepsilon + P + P - (\lambda_{\Pi\Pi} \Pi) - \frac{1}{2} \tau_{\Pi\Pi} (2\eta + \lambda_{\Pi\Pi} \Pi) - \frac{\tau_{\Pi\Pi}}{4} \Pi \geq 0.\]
Causality Violation in State-of-the-Art Simulations


arXiv: 2103.15889

See also C. Cheng and C. Shen

arXiv:2103.09848

Diagnostics:

Red: Acausal

Purple: Unknown

Blue: Causal

Scary

Pb+Pb collisions

- \( \tau = 1.16 \text{ fm/c} \)
- \( \tau = 2.02 \text{ fm/c} \)
- \( \tau = 2.92 \text{ fm/c} \)
- \( \tau = 3.81 \text{ fm/c} \)
- \( \tau = 4.71 \text{ fm/c} \)

- \( \tau = 0.40 \text{ fm/c} \)
- \( \tau = 1.40 \text{ fm/c} \)
- \( \tau = 2.40 \text{ fm/c} \)
- \( \tau = 3.40 \text{ fm/c} \)
- \( \tau = 4.40 \text{ fm/c} \)

- \( \tau = 0.80 \text{ fm/c} \)
- \( \tau = 1.80 \text{ fm/c} \)
- \( \tau = 2.80 \text{ fm/c} \)
- \( \tau = 3.80 \text{ fm/c} \)
- \( \tau = 4.80 \text{ fm/c} \)
Causality Violation in State-of-the-Art Simulations


arXiv: 2103.15889

~30% of initial cells acausal in state-of-the-art simulations

This issue must be fixed!

FIG. 2. Fractions of the total number of hydrodynamic cells ($e \geq e_{F0}$) in the causal (left), indeterminate (center), or acausal (right) categories, plotted as functions of the rescaled time evolution in each framework.
Let us now consider the other frontier

Neutron Star Mergers
Properties of matter at extreme baryon densities (neutron stars) remain unknown even in equilibrium.
Properties of matter at extreme baryon densities (neutron stars) remain unknown even in equilibrium.
Gravitationa Waves from Neutron Star Mergers

- Equation of state (not this talk)
- Probing viscous ultradense matter when gravity waves
How does a lump of baryon rich QCD matter flow under strong gravitational fields?

Viscous fluid dynamics + strong gravitational fields?

New signatures for deconfinement/phase transitions?
e.g. Most et al., PRL (2019)

Viscous effects in neutron star mergers?
Viscous effects in binary neutron-star mergers?


Shear dissipation: Relevant for trapped neutrinos if $T > 10$ MeV and gradients at small scales $\sim 0.01$ km (e.g., turbulence).

Thermal transport: Relevant for trapped electron neutrinos if $T > 10$ MeV and gradients $\sim 0.1$ km

“heat conductivity”
Viscous effects in binary neutron-star mergers?

Alford, Bovard, Hanauske, Rezzolla, Schwenzer, PRL (2018)

Bulk viscous damping in neutron star mergers

- Low densities (EOS)
- Neutrino transparency (low T)
- High frequencies (f > 1 kHz)

See also:

Alford, Harutyunyan, Sedrakian, PRD (2019); Alford, Haber, 2009.05181
Significant variations in:

- Temperature
- Density
- Fluid velocity

• Bulk viscosity effects should be investigated in simulations.

• Other viscous contributions deserve further investigation (e.g., thermal conductivity, shear viscosity).

Rezzolla group, Frankfurt

Alford et al. PRL (2018)
So, what will happen when we combine Ideal fluid, viscous fluid, and General Relativity?
Viscous Fluids in General Relativity
Why is this so challenging?

- Einstein + fluid equations are highly nonlinear.
- Previous approaches to viscous fluids are either acausal/unstable or not known to have a well-posed initial-value problem in relativity.
- The situation changed, dramatically, in the last couple of years, as I will now show you.
Generalized first-order theories of relativistic viscous hydrodynamics
Near Equilibrium Behavior:
The Derivative Expansion
Generalized First-Order Relativistic Hydrodynamics

Originally proposed by Bemfica, Disconzi, JN, PRD (2018): **Conformal regime**
Followed by Kovtun, JHEP (2019); Bemfica, Disconzi, JN, PRD (2019): **Non-conformal**
Hoult, Kovtun, JHEP (2020); Bemfica, Disconzi, JN, arxiv: 2009.11388: **Finite density**

In equilibrium: \( T^{\mu\nu} \rightarrow J^\mu \rightarrow T, \mu, u^\lambda \)

Well-defined Mapping

Temperature \( T \)

Chemical potential \( \mu \)

Flow velocity \( u^\lambda \)
Generalized First-Order Relativistic Hydrodynamics

Originally proposed by Bemfica, Disconzi, JN, PRD (2018): **Conformal regime**
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Out of equilibrium $T^{\mu\nu}$ $J^{\mu}$ are well defined

But hydrodynamic variables $T$, $\mu$, $u^{\lambda}$

can be defined in many ways

Different hydrodynamic frames

Eckart, Landau and Lifshitz + ...
Most general decomposition out of equilibrium

\[ T_{\mu\nu} = \mathcal{E} u_\mu u_\mu + \mathcal{P} \Delta_{\mu\nu} + \pi_{\mu\nu} + Q_\mu u_\nu + Q_\nu u_\mu \]

Conserved baryon current: \[ J_\mu = N u_\mu + J_\mu \]

Dissipation = more terms: \[ \mathcal{E}, N, \mathcal{P}, \pi_{\mu\nu}, Q_\mu, J_\mu \]

How does one fix such terms? Further assumptions needed.

- Standard approach: Derivative expansion
- Compatibility with the 2\textsuperscript{nd} law of thermodynamics.
- Recovers non-relativistic physics of fluids when \( v \ll 1 \).
Effective Field Theory Approach

• Write the most general expansion at 1st order in derivatives compatible with symmetries.

• Most general hydrodynamic frame.

$$\mathcal{E} = \varepsilon + \varepsilon_1 \frac{u^\alpha \nabla_\alpha T}{T} + \varepsilon_2 \nabla_\alpha u^\alpha + \varepsilon_3 u^\alpha \nabla_\alpha (\mu/T),$$

$$\mathcal{P} = P + \pi_1 \frac{u^\alpha \nabla_\alpha T}{T} + \pi_2 \nabla_\alpha u^\alpha + \pi_3 u^\alpha \nabla_\alpha (\mu/T),$$

$$\mathcal{N} = n + \nu_1 \frac{u^\alpha \nabla_\alpha T}{T} + \nu_2 \nabla_\alpha u^\alpha + \nu_3 u^\alpha \nabla_\alpha (\mu/T),$$

$$Q^\mu = \theta_1 \frac{\Delta^{\mu\nu} \nabla_\nu T}{T} + \theta_2 u^\alpha \nabla_\alpha u^\mu + \theta_3 \Delta^{\mu\nu} \nabla_\nu (\mu/T),$$

$$J^\mu = \gamma_1 \frac{\Delta^{\mu\nu} \nabla_\nu T}{T} + \gamma_2 u^\alpha \nabla_\alpha u^\mu + \gamma_3 \Delta^{\mu\nu} \nabla_\nu (\mu/T),$$

$$\pi_{\mu\nu} = -2\eta\sigma_{\mu\nu}$$

Usual transport coefficients

$$\eta, \zeta, \kappa$$

shear, bulk, heat cond.

New coefficients parametrize the freedom in choosing the hydro fields in this approach.

What choices are physical?
• In the regime of validity of the first-order theory, any choice of hydrodynamic frame satisfies the 2\textsuperscript{nd} law of thermo if $\eta, \zeta, \kappa \geq 0$

P. Kovtun, JHEP (2019)

\[
\begin{align*}
S^\mu &= (s + A - \mu N) u^\mu + \frac{Q^\mu - \mu J^\mu}{T} \\
\nabla_\mu S^\mu &= \text{positive} + O(\partial^3)
\end{align*}
\]

• New parameters can be constrained by causality and stability.


• This leads to a set of good definitions for $T, \mu, u^\lambda$
How can these new developments be used to understand the hot and viscous ultradense matter in neutron star mergers?

Figure by Most et al., PRL (2019)
Conserved current “a la Eckart”: \( J^\mu = n u^\mu \)

Energy-momentum tensor:

\[
T^{\mu\nu} = (\varepsilon + \mathcal{A}) u^\mu u^\nu + (P + \Pi) \Delta^{\mu\nu} - 2\eta \sigma^{\mu\nu} + u^\mu Q^\nu + u^\nu Q^\mu
\]

Out-of-equilibrium contributions

Contribution to energy density: \( \mathcal{A} = \tau_\varepsilon \left[ u^\lambda \nabla_\lambda \varepsilon + (\varepsilon + P) \nabla_\lambda u^\lambda \right] \)

Contribution to pressure: \( \Pi = -\zeta \nabla_\lambda u^\lambda + \tau_P \left[ u^\lambda \nabla_\lambda \varepsilon + (\varepsilon + P) \nabla_\lambda u^\lambda \right] \)

Energy flux: \( Q^\nu = \tau_Q (\varepsilon + P) u^\lambda \nabla_\lambda u^\nu + \beta_\varepsilon \Delta^{\nu\lambda} \nabla_\lambda \varepsilon + \beta_n \Delta^{\nu\lambda} \nabla_\lambda n \)
Causality

An essential property of relativity

Curved spacetime $\rightarrow$ Distorted light cone

General Relativity
System given by fluid + Einstein’s equations is causal

**Causality**

**Theorem 1.** Let \((\varepsilon, n, u^\mu, g_{\alpha\beta})\) be a solution to (2) and (12), with \(u^\mu u_\mu = -1\), defined in a globally spacetime \((M, g_{\alpha\beta})\). Assume that:

(A1) \(p = \varepsilon + P, \tau_\varepsilon, \tau_Q, \tau_P > 0\) and \(\eta, \zeta, \sigma \geq 0\).

Then, causality holds for \((\varepsilon, n, u^\mu, g_{\alpha\beta})\) if, and only if, the following conditions are satisfied:

\[
\rho \tau_Q > \eta,
\]

\[
\left[ \tau_\varepsilon \left( p c_s^2 \tau_Q + \zeta + \frac{4\eta}{3} + \sigma \xi_s \right) + \rho \tau_P \tau_Q \right]^2 \geq 4 \rho \tau_\varepsilon \tau_Q \left[ \tau_P \left( p c_s^2 \tau_Q + \sigma \xi_s \right) - \beta_\varepsilon \left( \zeta + \frac{4\eta}{3} \right) \right] \geq 0,
\]

\[
2 \rho \tau_\varepsilon \tau_Q > \tau_\varepsilon \left( p c_s^2 \tau_Q + \zeta + \frac{4\eta}{3} + \sigma \xi_s \right) + \rho \tau_P \tau_Q \geq 0,
\]

\[
\rho \tau_\varepsilon \tau_Q + \sigma \xi_s \tau_P > \tau_\varepsilon \left( p c_s^2 \tau_Q + \zeta + \frac{4\eta}{3} + \sigma \xi_s \right) + \rho \tau_P \tau_Q (1 - c_s^2) + \beta_\varepsilon \left( \zeta + \frac{4\eta}{3} \right).
\]

- Requires knowledge about the characteristics (nonlinear regime).
- Brute force determination of characteristics here is hopeless.
- Since our first work, we developed a new geometric approach to determine the characteristics and, hence, causality.
Well-Posedness
A system of PDEs is locally well-posed if:

• Given the initial data, a local solution exists.
• The local solution is unique.
• The solution depends continuously on initial data.

http://www.math.ucla.edu/~tao/Dispersive/
• Fluid + Einstein’s equations are causal and strongly hyperbolic, and the initial-value problem is well-posed.

Strong Hyperbolicity and Well-Posedness

Theorem II. Let \((\Sigma, \hat{g}_{\alpha \beta}, \hat{\kappa}_{\alpha \beta}, \hat{\varepsilon}, \hat{n}, \hat{n}, \hat{u}^\alpha, \hat{\nu}^\alpha)\) be an initial-data set for the system comprised of Einstein’s equation (1) and \(\nabla_\mu J^\mu = 0\), where \(T_{\alpha \beta}\) and \(J^\mu\) are given in (10). Assume that \(\hat{u}^\mu \hat{u}_\mu = -1, \hat{n} \geq C > 0\), where constant, and that \(\nabla_\mu J^\mu = 0\) holds for the initial data. Assume (A1) with \(\eta > 0\) and suppose that (20) of Th hold in strict form and that the transport coefficients are analytic functions of their arguments. Finally, assume \(\hat{g}_{\alpha \beta}, \hat{\varepsilon}, \hat{n}, \hat{u}^\alpha \in H^N(\Sigma)\) and that \(\hat{\kappa}_{\alpha \beta}, \hat{\varepsilon}, \hat{n}, \hat{u}^\alpha \in H^{N-1}(\Sigma), N \geq 5\), where \(H^N\) is the Sobolev space. Then, there exists a globally hyperbolic development of the initial data. This globally hyperbolic development is unique if taken to be the maximum globally hyperbolic development of the initial data.

• Nonlinear equations in first-order form \(\mathcal{A}^\alpha \partial_\alpha U = R\).

• Prove that all eigenvalues are real and the set of eigenvectors form a complete set.

• Combine this result with advanced PDE techniques.
Bemfica, Disconzi, JN, arxiv: 2009.11388

- Fluid + Einstein’s equations is causal and strongly hyperbolic, and the initial-value problem is well-posed.

**Strong Hyperbolicity and Well-Posedness**

**Theorem II.** Let $(\Sigma, \hat{g}_{\alpha\beta}, \hat{\kappa}_{\alpha\beta}, \tilde{\varepsilon}, \hat{n}, \hat{\nu}, \hat{u}^\alpha, \hat{\nu}^\alpha)$ be an initial-data set for the system comprised of Einstein’s eq. (1) and $\nabla_\mu J^\mu = 0$, where $T_{\alpha\beta}$ and $J^\mu$ are given in (10). Assume that $\hat{u}^\mu \hat{u}_\mu = -1$, $\hat{n} \geq C > 0$, where constant, and that $\nabla_\mu J^\mu = 0$ holds for the initial data. Assume (A1) with $\eta > 0$ and suppose that (20) of Th hold in strict form and that the transport coefficients are analytic functions of their arguments. Finally, assume $\hat{g}_{\alpha\beta}, \tilde{\varepsilon}, \hat{n}, \hat{u}^\alpha \in H^N(\Sigma)$ and that $\hat{\kappa}_{\alpha\beta}, \tilde{\varepsilon}, \hat{n}, \hat{u}^\alpha \in H^{N-1}(\Sigma)$, $N \geq 5$, where $H^N$ is the Sobolev space. Then, there exists a globally hyperbolic development of the initial data. This globally hyperbolic development is unique if taken to be the maximum globally hyperbolic development of the initial data.

- Equilibrium states are stable.

**Theorem III.** Let (72) have a set of $N$ linearly independent real eigenvectors $\{r_1, \ldots, r_N\}$. If (68) is causal and stable in the local rest frame $O$, then it is also stable in any other Lorentz frame $O'$ connected to $O$ by a Lorentz transformation.
Based on our experience with heavy-ions ...

**Ideal fluids** → **Viscous fluids**

*in flat spacetime*

Led to a paradigm shift and many new insights:

- Nearly perfect fluidity of the quark-gluon plasma
- Characterization of initial state (e.g. gluon saturation)
- Emergence of hydrodynamics far from equilibrium
- Connection to other fields: AdS/CFT, cold atoms
The inclusion of viscous effects in mergers will force us to go back to the drawing board.

NEW OPPORTUNITIES!!! NEW DISCOVERIES!!!
Conclusions

- QGP formed in heavy-ions forces us to explore relativistic fluids far from equilibrium.

- New constraints reveal challenges to far-from-equilibrium relativistic hydrodynamics.

- Neutron star mergers raise the possibility to determine the out-of-equilibrium properties of hot ultradense matter.

- New first-order formulation paves the way for describing for the first time all viscous fluids effects in general relativity.