Chirality and cosmology

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The gist

• Fundamental interactions are parity breaking

• maybe parity breaking is manifest in new cosmological physics (inflation, dark matter, dark energy, baryogenesis) as well?
Themes

Elegant mathematics

Proofs of principle

New observables

Some futuristic/academic
Some possibly observable
Subjects

• Circular polarization of the CMB
  • From density perturbations
  • From gravitational waves
  • From chiral gravitational-wave background

• Chiral gravitational waves
  • Pulsar timing arrays
  • Astrometry

• 21-cm polarization
collaborators

- CMB: Keisuke Inomata
- PTAs/astrometry: Selim Hotinli, Kim Boddy, Wenzer Qin, Liang Dai, Andrew Jaffe, Enis Belgacem
- 21-cm: Lingyuan Ji, Keisuke Inomata
- TAM formalism: Liang Dai, Donghui Jeong
Cosmic microwave background
Linearly polarized by anisotropic Thomson scattering
Circular polarization of CMB

- Does not arise at linear order in cosmological perturbations
  - *Linear* polarization from scattering of anisotropic radiation field
- Circular polarization arises at *second* order from photon-photon interactions

\[
\mathcal{L} = \frac{1}{2} (\mathbf{E}^2 - \mathbf{B}^2) + \frac{2\alpha^2}{45m^4} \left[ (\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2 \right]
\]

Anisotropic CMB background gives rise to anisotropic index of refraction that a given CMB photon passes through (Sawyer 2014; Montero-Camacho & Hirata, 2018)
But what about circular polarization

• Does not arise at linear order in cosmological perturbations
  (Thomson scattering induces only *linear* polarization)
But can be induced by propagation of linearly polarized light through birefringent medium

\[
P_{ab}(\hat{n}) = \frac{1}{\sqrt{2}} \begin{pmatrix} Q(\hat{n}) & U(\hat{n}) \\ U(\hat{n}) & -Q(\hat{n}) \end{pmatrix} \quad \Phi_{ab}(\hat{n}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_Q(\hat{n}) & \phi_U(\hat{n}) \\ \phi_U(\hat{n}) & -\phi_Q(\hat{n}) \end{pmatrix}
\]

\[
\phi_{Q,U}(\hat{n}) = \frac{2}{c} \int dr \, \omega(r)n_{Q,U}(\hat{n}r) \quad V(\hat{n}) = \epsilon_{ac} P^{ab}(\hat{n}) \Phi_{b}^{c}(\hat{n})
\]
Primordial density perturbations

\[ P_{ab}(\hat{n}) = \sum_{lm} P_{lm} Y_{(lm)ab}^{E}(\hat{n}) \quad \Phi_{ab}(\hat{n}) = \sum_{lm} \Phi_{lm} Y_{(lm)ab}^{E}(\hat{n}) \]

\[ V_{lm} = \sum_{l_1 m_1} \sum_{l_2 m_2} P_{l_1 m_1} \Phi_{l_2 m_2} \]

\[ \times \int d\hat{n} \epsilon^{ab}_{l_1 m_1} Y_{(l_1 m_1)ac}^{E}(\hat{n}) Y_{(l_2 m_2)b}^{E}(\hat{n}) Y_{lm}^{*}(\hat{n}) \]

\[ C_{l}^{VV} = \sum_{l_1 l_2} \frac{(2l_1 + 1)(2l_2 + 1)}{4\pi} \left[ C_{l_1}^{PP} C_{l_2}^{\Phi\Phi} - C_{l_1}^{P\Phi} C_{l_2}^{P\Phi} \right] (H_{l_1 l_2}^l)^2 \]
Now suppose primordial polarization or index-of-refraction tensor due to primordial GWs

- Polarization and index of refraction now have B (as well as E) modes:

\[
\sum_{l_1 l_2 \text{(odd)}} (2l_1 + 1)(2l_2 + 1) \frac{1}{4\pi} \left( (C_{l_1}^{PE} P_E^{l_2} \Phi_E^{l_2} - C_{l_1}^{PE} \Phi_E^{l_2} C_{l_2}^{PE} \Phi_E^{l_2}) + \left( C_{l_1}^{PB} P_B^{l_2} \Phi_B^{l_2} - C_{l_1}^{PB} \Phi_B^{l_2} C_{l_2}^{PB} \Phi_B^{l_2} \right) \right) |H_{l_1 l_2}^l|^2
\]

\[
+ \sum_{l_1 l_2 \text{(even)}} (2l_1 + 1)(2l_2 + 1) \frac{1}{4\pi} \left( C_{l_1}^{PE} P_E^{l_2} \Phi_B^{l_2} + C_{l_1}^{PB} P_B^{l_2} \Phi_E^{l_2} - 2C_{l_1}^{PE} \Phi_E^{l_2} C_{l_2}^{PB} \Phi_B^{l_2} \right) |H_{l_1 l_2}^l|^2
\]
Probably far from detectable

\[ \sqrt{\langle V^2 \rangle} \sim \begin{cases} 
8 \times 10^{-14} \text{ K} & \text{(for scalar perturbations),} \\
2 \times 10^{-17} \left( \frac{r}{0.06} \right) \text{ K} & \text{(for tensor perturbations).}
\end{cases} \]
Chiral photons from chiral GWs

$$\left\langle (\Delta V_{00})^2 \right\rangle^{1/2} \approx 1.5 \times 10^{-18} \left( \frac{r}{0.06} \right)^{1/2}$$

$$V_{00} = \frac{1}{\sqrt{4\pi}} \sum_{lm} (P_{lm}^{E} \Phi_{lm}^{B*} - P_{lm}^{B} \Phi_{lm}^{E*})$$

$$\langle V_{00} \rangle = \sum_{l} \frac{2l+1}{\sqrt{4\pi}} (C_{l}^{PE} \Phi_{l}^{B} - C_{l}^{PB} \Phi_{l}^{E})$$

$$\approx 2.6 \times 10^{-17} \Delta \chi \left( \frac{r}{0.06} \right),$$

$$\langle (\Delta V_{00})^2 \rangle = \sum_{l} \frac{2l+1}{4\pi} \left( C_{l}^{PE} C_{l}^{PB} \Phi_{l}^{B} \Phi_{l}^{E} + C_{l}^{PB} C_{l}^{PE} \Phi_{l}^{E} \Phi_{l}^{B} \right.$$

$$- 2C_{l}^{PE} \Phi_{l}^{E} C_{l}^{PB} \Phi_{l}^{B}$$

$$+ \left( C_{l}^{PE} \Phi_{l}^{B} \right)^2 + \left( C_{l}^{PB} \Phi_{l}^{E} \right)^2 - 2C_{l}^{PE} C_{l}^{PB} C_{l}^{PE} \Phi_{l}^{B} \Phi_{l}^{E} \right)$$
Pulsar timing arrays and gravitational waves
Consider polarized GW in $+z$ direction

$$z(\hat{n}) \propto (1 - \cos \theta) \cos 2\phi.$$  

$$z_{\ell m} = \int d\hat{n} Y_{\ell m}(\hat{n}) z(\hat{n}) \propto \sqrt{\frac{(2\ell + 1)(\ell - 2)!}{(\ell + 2)!}} (\delta_{m2} + \delta_{m,-2})$$  

$$C_\ell \propto (\ell - 2)!/(\ell + 2)!$$

$$\text{HD}(\Theta) = \sum_{\ell=2}^{\infty} (2\ell + 1) \frac{(\ell - 2)!}{(\ell + 2)!} P_\ell(\cos \Theta)$$

$$= \frac{1}{2} (1 - x) \log \left[ \frac{1}{2} (1 - x) \right] - \frac{1}{6} \left[ \frac{1}{2} (1 - x) \right] + \frac{1}{3}$$
\[(\delta n)_a(\hat{n}) = \sum_{lm} \left[ E_{lm} Y_{(lm)}^E(\hat{n}) + B_{lm} Y_{(lm)}^B(\hat{n}) \right] \]
Total angular momentum waves

- Standard approach

\[
\phi(\mathbf{x}) = \sum \tilde{\phi}(\mathbf{k}) e^{i \mathbf{k} \cdot \mathbf{x}}
\]

\[
\phi(\mathbf{x}) = \sum_{klm} \phi_{klm} \Psi_{klm}(\mathbf{x})
\]

\[
\Psi_{klm}(\mathbf{x}) = 4\pi i^l j_l(kr) Y_{lm}(\hat{r})
\]
• Analogous expansions for vector fields in terms of three types (E, B, L) of vector TAM waves

• Analogous expansions for STF tensor fields in terms of 5 types (L, VE, VB, TE, TB) of tensor TAM waves. Transverse-traceless are TE/TB

• TAM formalism far more powerful for vector/tensor fields (than for scalar) given the transformation properties (the “spin”) of these fields under rotations
For PTA/astrometry

• Is trivial to consider contribution of any given TAM wave to PTA/astrometry observables

\[ C_{\ell}^{XX',\alpha} = 32\pi^2 F_{\ell}^{X,\alpha} \left( F_{\ell}^{X',\alpha} \right)^* \]

\[ \times \int df \frac{6 H_0^2 \Omega_{\alpha}(f)}{(2\pi)^3 f^3} W_X(f) W_{X'}(f) \]
<table>
<thead>
<tr>
<th>$X$</th>
<th>$F_{\ell}^{E,X}$</th>
<th>$F_{\ell}^{B,X}$</th>
<th>$F_{\ell}^{z,X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ST$</td>
<td>$\frac{i}{6} \delta_{\ell_1}$</td>
<td>0</td>
<td>$-\frac{1}{2\sqrt{2}} \left( \delta_{\ell_0} + \frac{i}{3} \delta_{\ell_1} \right)$</td>
</tr>
<tr>
<td>$SL$</td>
<td>$-\frac{i}{3\sqrt{2}} \delta_{\ell_1} + \frac{i^\ell}{2\sqrt{\ell(\ell + 1)}}$</td>
<td>0</td>
<td>$-\frac{i^\ell}{4} \ln(kr_s)$</td>
</tr>
<tr>
<td>$VE$</td>
<td>$\frac{2i}{3\sqrt{2}} \delta_{\ell_1} - \frac{i^\ell}{\sqrt{2\ell(\ell + 1)}}$</td>
<td>0</td>
<td>$-\frac{i}{3} \delta_{\ell_1} + \frac{i^\ell}{\sqrt{2\ell(\ell + 1)}}$</td>
</tr>
<tr>
<td>$VB$</td>
<td>0</td>
<td>$\frac{i}{3\sqrt{2}} \delta_{\ell_1} - \frac{i^\ell}{\sqrt{2\ell(\ell + 1)}}$</td>
<td>0</td>
</tr>
<tr>
<td>$TE$</td>
<td>$-\frac{i^\ell}{\sqrt{\ell(\ell + 1)}} \frac{N^{-1}_{\ell}}{}$</td>
<td>0</td>
<td>$\frac{i^\ell}{2} N^{-1}_{\ell}$</td>
</tr>
<tr>
<td>$TB$</td>
<td>0</td>
<td>$-\frac{i^\ell}{\sqrt{\ell(\ell + 1)}} \frac{N^{-1}_{\ell}}{}$</td>
<td>0</td>
</tr>
</tbody>
</table>
GW anisotropy with PTAs

\[ \langle h_s(\vec{k}) h^*_s(\vec{k}') \rangle = \frac{1}{4} \delta_{ss'} (2\pi)^3 \delta_D(\vec{k} - \vec{k}') P_h(k) \]

\times \left[ 1 + \sum_{L>0} \sum_{M=-L}^{L} g_{LM} Y_{LM}(\hat{k}) \right] \]

\[ \langle z_{\ell m} z^*_{\ell' m'} \rangle = C_\ell \delta_{\ell \ell'} \delta_{m m'} \]

\[ + \sum_{L=1}^{\infty} \sum_{M=-L}^{L} (-1)^{m'} \langle \ell m \ell', -m' \mid LM \rangle A^{LM}_{\ell \ell'} \]
\[ z_{\ell m}(\hat{z}) = z_{\ell} \left[ h_{+}(\delta_{m2} + \delta_{m,-2}) + i h_{\times}(\delta_{m2} - \delta_{m,-2}) \right] \]
\[ = z_{\ell} \left[ (h_{+} + i h_{\times})\delta_{m2} + (h_{+} - i h_{\times})\delta_{m,-2} \right], \]

\[ z_{\ell m}(\hat{k}) = \sum_{m'} D_{mm'}^{(\ell)}(\phi_k, \theta_k, 0) z_{\ell m'}(\hat{z}) \]
\[ = z_{\ell} \left[ (h_{+} + i h_{\times})D_{m2}^{(\ell)} + (h_{+} - i h_{\times})D_{m,-2}^{(\ell)} \right] \]

\[ z_{\ell m} = \sqrt{2} z_{\ell} \int \frac{d^3 k}{(2\pi)^3} \left[ h_R(\vec{k})D_{m2}^{(\ell)} + h_L(\vec{k})D_{m,-2}^{(\ell)} \right] \]

\[ \langle z_{\ell m} z_{\ell' m'}^{*} \rangle = z_{\ell} z_{\ell'} \int \frac{d^3 k}{(2\pi)^3} P_h(k) \left[ 1 + \sum_{LM} g_{LM} Y_{LM}(\hat{k}) \right] \]
\[ \times \left[ D_{m2}^{(\ell)}(\hat{k}) \left( D_{m'2}^{(\ell')}(\hat{k}) \right)^* + D_{m,-2}^{(\ell)}(\hat{k}) \left( D_{m',-2}^{(\ell')}(\hat{k}) \right)^* \right]. \quad (20) \]
\[ C_\ell = \frac{z_\ell^2}{2\ell + 1} I, \]

\[ A_{\ell \ell'}^{LM} = (-1)^{\ell - \ell'} (4\pi)^{-1/2} g_{LM} z_\ell z_{\ell'} H_{\ell \ell'}^{L} I, \]

\[ (\text{SNR})^2 = \sum_{\ell = 2}^{\ell_{\text{max}}} \frac{(2\ell + 1)}{2} \left( \frac{C_\ell}{N_{zz}} \right)^2 \]

\[ (\Delta g_{LM})^{-2} = \frac{27}{16\pi^3} \sum_{\ell \ell'} \frac{[H_{\ell \ell'}^{L} z_\ell z_{\ell'} (\text{SNR}) N_{zz}]^2}{(C_\ell + N_{zz})(C_{\ell'} + N_{zz})} \]
Bottom line

- Isotropic signal must be established with high SNR before anisotropy can be detected, and then only if anisotropy amplitude is $O(1)$. 
Back to chirality!!

- Anisotropy estimator easily modified to seek GW circular-polarization

\[
\begin{align*}
\langle h_R(\vec{k}) h_R^*(\vec{k}') \rangle &= \frac{1}{4} (2\pi)^3 \delta_D(\vec{k} - \vec{k}') P_h(k) \left[ 1 - \epsilon(\hat{k}) \right] , \\
\langle h_L(\vec{k}) h_L^*(\vec{k}') \rangle &= \frac{1}{4} (2\pi)^3 \delta_D(\vec{k} - \vec{k}') P_h(k) \left[ 1 + \epsilon(\hat{k}) \right] , \\
\langle h_R(\vec{k}) h_L^*(\vec{k}') \rangle &= 0 .
\end{align*}
\]

\[
\epsilon(\hat{k}) = \sum_{L=0}^{\infty} \sum_{M=-L}^{L} \epsilon_{LM} Y_{LM}(\hat{k})
\]
\[
\begin{align*}
  z_{\ell m}(\hat{k}) &= \sum_{m'} D_{mn}^{(\ell)}(\phi_k, \theta_k, 0) z_{\ell m'}(\hat{z}) \\
  &= z_\ell \left[ (h_+ + i h_\times) D_{m_2}^{(\ell)} + (h_+ - i h_\times) D_{m_2}^{(\ell)} \right] \\
  z_{\ell m} &= \sqrt{2} z_\ell \int \frac{d^3 k}{(2\pi)^3} \left[ h_R(\vec{k}) D_{m_2}^{(\ell)} + h_L(\vec{k}) D_{m_2}^{(\ell)} \right] \\
  \langle z_{\ell m} z_{\ell' m'}^* \rangle &= \frac{1}{8} z_\ell z_{\ell'} \int \frac{d^3 k}{(2\pi)^3} P_h(k) \left\{ D_{m_2}^{(\ell)}(\hat{k}) \left( D_{m_2}^{(\ell')}(\hat{k}) \right)^* \right. \\
  &\times \left[ 1 + \sum_{LM} \epsilon_{LM} Y_{LM}(\hat{k}) \right] + D_{m_2}^{(\ell)}(\hat{k}) \left( D_{m_2}^{(\ell')}(\hat{k}) \right)^* \\
  &\times \left[ 1 - \sum_{LM} \epsilon_{LM} Y_{LM}(\hat{k}) \right] \right\}. 
\end{align*}
\]
• Anisotropy -→ CP :: + → -

• $L+l+l'$ = even → $L+l+l'$ = odd

• So estimators for intensity anisotropy become estimators for circular-polarization anisotropy by placing $L+l+l'$ even to $L+l+l'$ odd

• One consequence: monopole (an overall GW chirality) not detectable

• Numerically, CP dipole detectable with high SNR and for CP dipole $O(1)$
This is interesting!

- ~nHZ GW background from SMBH-binary mergers
- Local signal may well be dominated by one, or a handful of sources, and if so, intensity should be anisotropic, and signal most generally circularly polarized to O(1)
Circular polarization of 21-cm radiation from dark ages

• Hirata, Mishra & Venumadhav (2017) calculated circular polarization from 21-cm line of neutral hydrogen from misalignment of 21-cm quadrupole and CMB quadrupole. Calculation performed with spherical tensors.

• Our work (in progress): reformulate in terms of Cartesian tensors and then use TAM to simplify calculation. Provide first numerical results on “standard model” prediction (that arises from 2nd order in density-perturbation amplitude)
neutral hydrogen (with spin)
density perturbation $\delta$

last-scattering surface
density perturbation $\delta$

spin quadrupole $\gamma_{ab}$

CMB quadrupole $t_{ab}$

circularly polarized 21-cm photons

$V \propto \epsilon_{abc} n_a \gamma_{bd} t_{cd}$

quadrupole-quadrupole interaction

[Hirata+ 1707.07513] [LJ, Kamionkowski, Inomata in prep.]
21cm Circular Polarization: Results

\[ C_l^{VV} = |C|^2 \sum_{J_\gamma, J_t (\text{odd})} \sum_{k_\gamma k_t} P(k_\gamma) P(k_t) T_\gamma(k_\gamma) T_t(k_t) \left[ T_\gamma(k_\gamma) T_t(k_t) - T_\gamma(k_t) T_t(k_\gamma) \right] \]

\[ \times \frac{(2J_\gamma + 1)(2J_t + 1)}{4\pi} \left| \tilde{R}_{J_\gamma}^{VE}(k_\gamma \chi) \tilde{R}_{J_t}^{VE}(k_t \chi) \begin{pmatrix} l & J_\gamma & J_t \\ 0 & +1 & -1 \end{pmatrix} - \tilde{R}_{J_\gamma}^{TE}(k_\gamma \chi) \tilde{R}_{J_t}^{TE}(k_t \chi) \begin{pmatrix} l & J_\gamma & J_t \\ 0 & +2 & -2 \end{pmatrix} \right|^2 \]


[LJ, Kamionkowski, Inomata in prep.]
21cm Circular Polarization: Results

$C_{\ell}^{VV}$ vs $\ell$

$LJ, Kamionkowski, Inomata in prep.$

[Kovetz+ 1807.11482]
Numerically....

- Signal far too big to be seen any time soon, but conceivably detectable with future lunar-based radio array
- May be interesting cross-correlations with other observables (CMB B mode, weak lensing, etc.)
Summary/conclusions

• Calculational tools from TAM formalism allow dramatic simplification of observables on a spherical sky, especially when vector/tensor modes are involved and/or observables are 2nd order in perturbation theory
  • CMB circular polarization in standard model
  • Imprint of chirality of GW background on CMB CP
  • Elegant/compact formalism for PTA/astrometry probes of ~nHZ GW background
  • CP of 21-cm radiation from dark ages