Astrophysical constraints on the high-density equation of state

Sophia Han, TDLI Fellow
T.D. Lee Institute, Shanghai Jiao Tong Univ.
Outline

- Introduction - dense matter and neutron stars

- Neutron star structure and the equation of state (EoS)

- Multi-messenger constraints on the EoS: what have we learned so far?

- Future directions
QCD phase diagram

- lattice QCD gives good result at finite temperature, but is stymied currently at finite density
- perturbative QCD: only valid at asymptotically high densities
- can’t calculate properties of cold dense matter, must observe!

- properties of ultra-dense matter in the inner cores of neutron stars (NSs)
- a challenging problem as no terrestrial experiments can probe such high densities
- also because reliable first-principle calculations break down at the strongly-interacting regime

3G Science white paper
Dense matter in NSs

- stable nuclei
- neutron-rich nuclei
- neutron-rich nuclei with quasi-free neutrons
- homogeneous nucleonic matter (liquid)
- exotica

Fundamental questions

- what are the most relevant lower-energy degrees of freedom?
- how does deconfinement evolve as $T \to 0$ on the QCD phase diagram?
Nature’s extreme labs

• for the interior of a spherical, static, relativistic star

\[
\frac{dp}{dr} = -\varepsilon(r) \frac{Gm(r)}{r^2} \left[ 1 + \frac{p(r)}{\varepsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 p(r)}{m(r)} \right] \left[ 1 - \frac{2Gm(r)}{r} \right]^{-1}
\]

\[m(r) \equiv 4\pi \int_0^r \varepsilon(r) r^2 dr\]

massive neutron stars \(\sim 2M_\odot\) do exist!

<table>
<thead>
<tr>
<th>Source</th>
<th>Mass (M_\odot)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSR J1614-2230</td>
<td>1.97 ± 0.04</td>
<td>Demorest et al. (2010)</td>
</tr>
<tr>
<td></td>
<td>1.928 ± 0.017</td>
<td>Fonseca et al. (2016)</td>
</tr>
<tr>
<td></td>
<td>1.908 ± 0.016</td>
<td>Arzoumanian et al. (2018)</td>
</tr>
<tr>
<td>PSR J0438+0432</td>
<td>2.01 ± 0.04</td>
<td>Antoniadis et al. (2013)</td>
</tr>
<tr>
<td>PSR J0740+6620</td>
<td>2.14^{+0.10}_{-0.09}</td>
<td>Cromartie et al. (2019)</td>
</tr>
<tr>
<td></td>
<td>2.08 ± 0.07</td>
<td>Fonseca et al. (2021)</td>
</tr>
</tbody>
</table>
NS mass-radius diagram

massive neutron stars $\sim 2M_\odot$ do exist!
self-bound stars with a bare surface e.g. strange matter hypothesis

continuous (and mostly smooth) profile for normal hadronic EoSs; *also possible with weak/mild phase transition or crossover

substantial softening e.g. discontinuity in the energy density induced by a strong sharp phase transition
Schematic EoSs from theory

- self-bound stars with a bare surface e.g. strange matter hypothesis
- continuous (and mostly smooth) profile for normal hadronic EoSs; *also possible with weak/mild phase transition or crossover
- substantial softening e.g. discontinuity in the energy density induced by a strong sharp phase transition

\[ c_s^2 = \frac{d p}{d \varepsilon} \]
From nuclei to neutron stars


\[ P(\varepsilon) = n^2 \frac{\partial E_n(n)}{\partial n} \]

\[ L = 3n_{\text{sat}} S'(n_{\text{sat}}) \]

\[ S = 32 \pm 5 \text{ MeV} \]

\[ n_{\text{sat}} \approx 0.16 \text{ fm}^{-3} \]

- nuclear experiments correlate S and L
- theory extrapolates in isospin and baryon density

stable nuclei is here
[sself-bound system]
From nuclei to neutron stars

PREX Collaboration, arXiv: 2102.10767

new! neutron skin measurement @JLab

- under hot debate: pressures at sub-nuclear densities and the inferred value of $L$

PREX Collaboration, arXiv: 2102.10767
Nuclear physics input

- Pressure at low densities (outer core) controls typical NS radii: stiff or soft?
- Reliably quantified uncertainties from chEFT for beta-equilibrated NSM
- Less than ~5% deviation from PNM pressures
- To extrapolate or match at higher densities in the inner core

\[ E_{\text{NSM}} = E_{\text{PNM}} (1 - 2x)^2 + E_{\text{SNM}} 4x (1 - x) + .. + E_e + E_{\mu} \]

\[ x = \frac{n_p}{n_B} \leq 5.5\% \]

\[ \frac{\partial E_{\text{NSM}}}{\partial x} = 0 \]

\[ n_{\text{sat}} \approx 0.16 \text{ fm}^{-3} \]
Typical scenarios

- nuclear matter only (standard)
- 1st-order PT
- crossover/quarkyonic
- sharp boundary - Maxwell
- mixed phase - Gibbs (geometrically separated)

- masquerade problem: likely indistinguishable through observations that constrain M-R only
- smooth crossover: no more easily understood in terms of hadrons than in terms of quarks
- 1st-OPT: mixed phase (Gibbs) is favored if the hadron/quark surface tension is small
X-ray probes of NS radii

- conventional methods of radius estimates through surface photon emission detection suffer from large uncertainties
  Ozel & Freire (2016); Steiner et. al (2016)

- photospheric radius expansions (PREs)
  - quiescent low-mass x-ray binaries (QLMXBs)
First BNS merger detection

GW170817 that unveiled the multi-messenger era

- “hear” cosmic collisions between densest astronomical objects
- follow-up E&M signals; “see” e.g. evidence for nucleosynthesis

EoS affects GW emission during inspiral

- tidal deformability
  \[ \Lambda \equiv \frac{\lambda}{M^5} \equiv \frac{2}{3} k_2 \left(\frac{Rc^2}{GM}\right)^5 \]
- compactness
  \[ \beta \equiv \frac{M}{R} \]

\[ Q_{ij} = -\lambda \varepsilon_{ij} \]
Impact on pre-merger GW signal

- tidal Love number depends on the EoS and the compactness $M/R$
- matter effects (NSs) leave imprints in the waveform - distinguish from point-particles (BHs)
- much cleaner systematics

larger tidal parameter $\leftrightarrow$ faster GW emission

Postnikov et al. (2010)
Hinderer et al. (2010)

Chatziioannou
arXiv:2006.03168
Measuring NS radius with GWs

\[ m_{\text{tot}} = 2.73^{+0.04}_{-0.01} M_\odot \]

\[ \mathcal{M} = \left( \frac{m_1 m_2}{m_1 + m_2} \right)^{3/5} = 1.186^{+0.001}_{-0.001} M_\odot \]

LVC collaboration, arXiv:1805.11579

- smaller R (favored) <13.6km
- larger R (disfavored)

chirp mass extremely well measured

effect from mass ratio uncertainty is small
Pure neutron matter (PNM)

low-density matter is soft-ish

$P \pm 1\sigma$ [MeV fm$^{-3}$]

$c_s^2 \pm 1\sigma$ [\(\frac{dP}{d\varepsilon}\)]

$0.1$  $0.2$  $0.3$

Density $n$ [fm$^{-3}$]

$0.1$  $0.2$  $0.3$

Density $n$ [fm$^{-3}$]

BUQEYE Collab.
(2020)
GW + heavy pulsars

- sound speed in the **core** and **when** rapid stiffening in the EoS begins

\[ \frac{n_m}{n_{\text{sat}}} \]

\[ c_s^2 \approx \frac{1}{3} \]

\[ \tilde{\Lambda}_{1.186} \leq 720 \]

GW190814

- extremely loud event produced by the inspiral and merger of two compact objects -- one, a black hole, and the other of undetermined nature
- the mass measured for the lighter compact object makes it either the lightest black hole or the heaviest neutron star ever discovered

\[ m_1 = 23.2^{+1.1}_{-1.0} \, M_\odot \quad \text{and} \quad m_2 = 2.59^{+0.08}_{-0.09} \, M_\odot \]

LVC collaboration, arXiv:2006.12611
Sound speed in the core

\[ c_s^2(r) \equiv \frac{dp(r)}{d\varepsilon(r)} \]

how fast pressure rises with energy density

Possible behavior in neutron star interiors

- minimal scenario of normal nuclear matter: (smoothly) continuous function of pressure
- first-order phase transition scenario: finite energy density discontinuity induces sudden softening near the phase boundary
- crossover scenario/quarkyonic matter

Limits

- asymptotically high density: \( \sim 1/3 \)
- \( \sim 4-8 \) times saturation: supports massive NSs
- high-T: matches lattice calc./heavy-ion data

McLerran & Reddy,
PRL 122, 122701 (2019)
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Baym et al. arXiv:1707.04966
Rept. Prog. Phys. 81, 056902
Constraints from max. mass

DBHF (stiff) NM, $c_{QM}^2 = 1/3$

- with weakly interacting quarks, very limited to reach two solar masses
- high transition density scenario: resembles no PT; short extension
- low transition density scenario: no twin stars

\[ \varepsilon(p) = \varepsilon_{\text{trans}} + \Delta\varepsilon + c_{QM}^{-2}(p - p_{\text{trans}}) \]

still survives the conformal limit
e.g. a population of BNS events

Chatziioannou & SH, arXiv:1911.07091
SH & Steiner, arXiv:1810.10967

• might identify third-family stars [strong 1st-OPT] with pre-merger GWs

• requires multiple (N~50-100) future detections to separate different families: NS-NS, NS-HS, HS-HS mergers

\[
\frac{\Delta \varepsilon_{\text{crit}}}{\varepsilon_{\text{trans}}} = \frac{1}{2} + \frac{3 p_{\text{trans}}}{2 \varepsilon_{\text{trans}}} \quad \text{Seidov (1971)}
\]
e.g. a population of BNS events

\[ \Lambda \equiv \frac{\lambda}{M^5} \equiv \frac{2}{3} k^2 \left( \frac{R c^2}{GM} \right)^5 \]

Chatziioannou & SH, arXiv:1911.07091

SH & Steiner, arXiv:1810.10967

most populated if the normal branch > 13 km and the high density matter is still strongly interacting \( c_s^2 \gtrsim 0.4 \)

- might identify third-family stars [strong 1st-OPT] with pre-merger GWs
- requires multiple (N~50-100) future detections to separate different families: NS-NS, NS-HS, HS-HS mergers
NS radii from hotspots

- light-curve modeling of x-ray pulse profiles that are sensitive to the stellar compactness $M/R$

*Neutron star Interior Composition ExploreR*

- most recent data on the heaviest NS known so far: combined information with precise mass measurements through Shapiro delay (radio)

Cromartie et al.
Independent NICER team analyses

Riley et al. 2019

M = 1.34\,^{+0.15}_{-0.16}\, M_\odot
R = 12.71\,^{+1.14}_{-1.19}\, km

Miller et al. 2019

M = 1.44\,^{+0.15}_{-0.14}\, M_\odot
R = 13.02\,^{+1.24}_{-1.06}\, km

Credit: NASA's Goddard Space Flight Center/CI Lab

PSR J0030+0451
Results published together in an ApJ Letters Focus Issue in December 2019
NS radii from hotspots

new! radius of PSR J0740+6620 \( \sim 2 \, M_\odot \, 13.7^{+2.6}_{-1.5} \) km vs. \( 12.4^{+1.3}_{-1.0} \) km

previously: PSR J0030+0451 \( \sim 1.4 \, M_\odot \)

• analyses of waveforms produced by hotspots of rotation-powered pulsars
• tend to favor relatively stiffer EoS at intermediate \( (2 \sim 3 \, n_{\text{sat}}) \) densities
Multimessenger constraints

$R_{2.0} = 12.2 \pm 1.2 \text{ km}$

$R_{1.4} = 12.32 \pm 1.23 \text{ km}$

- nonparametric survey conditioned on ensembles of existing model EoSs
- GW170817+190425, NICER J0030 & J0740, and massive pulsars
• tightening the pressure constraint at intermediate densities
• (90% symmetric credible intervals) best compatibility with data
Single branch vs. multiple branches

- full posterior is dominated by EoSs with a single stable branch
- onset for the unstable branch (extra softening) pushed to two ends

expected from max. mass
Summary

- multimessenger constraints point to NS radii around 12.5 km $\pm$ 1.5 km

- most extreme phase transitions that lead to drastic $>$2-3 km reduction seem disfavored; onset restricted to either low or high densities

- milder PTs or smooth crossovers are fairly consistent with data; requires high sound speed in the inner core

- pressure or stiffness in nuclear EoS up to twice saturation density is crucial for interpretations of high-density behavior: the golden window
Looking forward

GW190425

• total mass ~3.4 solar masses
• signal too weak to provide further EoS constraints R<16 km

GW190814

• component of ambiguous nature
• most asymmetric system observed

see events of GWTC-2: arXiv:2010.14527

more mass-gap objects?

LVC collaboration

LVC collaboration
arXiv:2006.12611
Fate of merger remnant

- GW + EM constraints from 170817 seem to favor $M_{\text{max}} < 2.16 \sim 2.3$ solar masses. Ruiz et al. (2018), Rezzolla et al. (2018), Shibata et al. (2019)

- radius >10.68 km to prevent prompt collapse. Bauswein et al. (2017)
Post-merger dynamics

- complicated spectra of excited modes depend on the EoS
- location of the dominant peak strongly correlated with NS radii
- within reach of next generation GW detectors (~10 times more sensitive)

Bauswein & Stergioulas, arXiv:1502.03176
Takami et al., arXiv:1412.3240
e.g. softening effects on post-merger GW

- hyperon onset
- more compact remnant (higher central density)
- earlier collapse; higher frequency

Most et al. arXiv:1807.03684
PRL 122, 061101 (2019)

Radice et al. arXiv:1612.06429

1st-OPT to soft quark matter after merger
e.g. softening effects on post-merger GW

third-family stars

- **stiff** EoS at low density - DD2
- **strong 1st-OPT** to **stiff quark matter**

Fujimoto et al. (2022)

Bauswein & Blacker (2020)

- **soft** EoS at low density \( \sim \) N3LO chiEFT
- **rapid stiffening** in the crossover regime

**crossover to soft quark matter after merger**
NSBH mergers

Observation of Gravitational Waves from Two Neutron Star–Black Hole Coalescences

**Table 2**
Source Properties of GW200105 and GW200115

<table>
<thead>
<tr>
<th></th>
<th>GW200105</th>
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<th>GW200115</th>
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<tbody>
<tr>
<td></td>
<td>Low Spin</td>
<td>High Spin</td>
<td>Low Spin</td>
</tr>
<tr>
<td>Primary mass $m_1/M_\odot$</td>
<td>8.9$^{+1.1}_{-1.3}$</td>
<td>8.9$^{+1.2}_{-1.3}$</td>
<td>5.9$^{+1.1}_{-1.2}$</td>
</tr>
<tr>
<td>Secondary mass $m_2/M_\odot$</td>
<td>1.9$^{+0.2}_{-0.3}$</td>
<td>1.9$^{+0.3}_{-0.2}$</td>
<td>1.4$^{+0.2}_{-0.3}$</td>
</tr>
<tr>
<td>Mass ratio $q$</td>
<td>0.2$^{+0.06}_{-0.04}$</td>
<td>0.2$^{+0.06}_{-0.04}$</td>
<td>0.24$^{+0.31}_{-0.09}$</td>
</tr>
<tr>
<td>Total mass $M/M_\odot$</td>
<td>10.8$^{+0.9}_{-1.0}$</td>
<td>10.9$^{+0.9}_{-1.0}$</td>
<td>7.3$^{+1.2}_{-1.3}$</td>
</tr>
<tr>
<td>Chirp mass $\mathcal{M}/M_\odot$</td>
<td>3.4$^{+0.08}_{-0.07}$</td>
<td>3.4$^{+0.08}_{-0.07}$</td>
<td>2.42$^{+0.05}_{-0.07}$</td>
</tr>
<tr>
<td>Detector-frame chirp mass $(1+z)\mathcal{M}/M_\odot$</td>
<td>3.619$^{+0.006}_{-0.006}$</td>
<td>3.619$^{+0.006}_{-0.006}$</td>
<td>2.580$^{+0.006}_{-0.007}$</td>
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<tr>
<td>Primary spin magnitude $\chi_1$</td>
<td>0.09$^{+0.08}_{-0.12}$</td>
<td>0.08$^{+0.08}_{-0.12}$</td>
<td>0.31$^{+0.52}_{-0.29}$</td>
</tr>
<tr>
<td>Effective inspiral spin parameter $\chi_{\text{eff}}$</td>
<td>$-0.01^{+0.08}_{-0.12}$</td>
<td>$-0.01^{+0.08}_{-0.12}$</td>
<td>$-0.14^{+0.17}_{-0.34}$</td>
</tr>
<tr>
<td>Effective precession spin parameter $\chi_p$</td>
<td>0.07$^{+0.06}_{-0.10}$</td>
<td>0.06$^{+0.06}_{-0.10}$</td>
<td>0.19$^{+0.28}_{-0.17}$</td>
</tr>
<tr>
<td>Luminosity distance $D_L$/Mpc</td>
<td>280$^{+110}_{-110}$</td>
<td>280$^{+110}_{-110}$</td>
<td>310$^{+50}_{-10}$</td>
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<tr>
<td>Source redshift $z$</td>
<td>0.06$^{+0.02}_{-0.02}$</td>
<td>0.06$^{+0.02}_{-0.02}$</td>
<td>0.07$^{+0.03}_{-0.02}$</td>
</tr>
</tbody>
</table>

- GW200105: $\sim 1.9 + \sim 9$ solar masses
- GW200115: $\sim 1.5 + \sim 6$ solar masses

no information on matter effects
no significant EM detections

see events of GWTC-3: arXiv:2111.03606
• NS is either tidally disrupted or plunges into the BH - mass ratio, spin, EoS
• radius determines if tides are measurable & if EM signals can be produced
More opportunities

probing dense matter in NSs

- cooling of NS 1987A - neutrino emissivity, stellar superfluids (nuclear theory, condensed matter)
- merger evolution and astro/GW signals - out-of-equilibrium physics; composition details (simulation, nucleosynthesis)
- next Galactic supernova? (neutrino physics)
- asteroseismology (hydrodynamics, GR, nucl-th)
- …and more - add your own!

Rev. Mod. Phys. 88, 021001 (2016)
THANK YOU!

Q & A