Chiral anomalous processes in magnetospheres of compact stars

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September 29, 2021
Spin state and momentum are independent quantities for non-relativistic electrons.

In the ultrarelativistic limit, the Weyl equation

\[ \mathcal{H}_W = \pm c \sigma \cdot k \]

implies that spin is completely locked to momentum.

**Right-handed (R) fermion**

**Left-handed (L) fermion**

Electric and chiral currents

\[ j = j_R + j_L, \quad j_5 = j_R - j_L \]
In a magnetic field, LLL states are completely spin polarized. Right- and left-handed electrons propagate in the opposite directions → an imbalance (quantified by $\mu_5$) results in an electric current $j = e^2 \mu_5 B / (2\pi^2)$. This is the chiral magnetic effect (CME).
Parallel electric and magnetic fields break a balance between the Fermi surfaces of the right- and left-handed fermions

\[ \dot{k}_R = eE \rightarrow \frac{dN_R}{dt \; dz} = \frac{eE}{2\pi}, \]  

\[ \frac{dN_R}{dx \; dy} = \frac{eB}{2\pi} \rightarrow \frac{dN_R}{dt \; dV} = \frac{e^2(E \cdot B)}{(2\pi)^2}, \]  

\[ \frac{dN_L}{dt \; dV} = -\frac{dN_R}{dt \; dV} \]
Therefore, the chiral charge is not conserved (the chiral anomaly) [S. L. Adler, Phys. Rev. 177, 2426, (1969); J. S. Bell and R. Jackiw, Nuovo Cim. A 60, 47, (1969)]

\[ \dot{q}_5 \equiv \dot{n}_R - \dot{n}_L = \frac{e^2 (E \cdot B)}{2\pi^2} \]  

Figure: Chiral imbalance \( \mu_5 = (\mu_R - \mu_L)/2 \neq 0 \) induced by the chiral anomaly
Many-body chiral fermion systems

Chiral matter is realized in the following physical systems:

- Ultrarelativistic primordial plasma in early Universe
- Quark-gluon plasma in heavy-ion collisions
- Electron quasiparticles in Dirac and Weyl semimetals
- Degenerate electrons in compact stars
- Relativistic jets in black holes and neutron stars

Is there a chiral asymmetry in relativistic jets?
Chirality production in proto-neutron stars

Electron capture (core collapse $10^6 \text{ km} \rightarrow 10 \text{ km}$)

$$p + e^-_L \rightarrow n + \nu^e_L$$

is a weak interaction process where left-handed electrons are captured by protons producing $\mu_5 \neq 0$. A. Ohnishi and N. Yamamoto, arXiv:1402.4760 suggested that magnetic field of neutron stars can be generated due to the chiral magnetic instability. Maxwell‘s equations

$$\nabla \times B = j + \frac{\partial E}{\partial t}, \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

with the CME and Ohm‘s currents

$$j = \frac{\mu_5 B}{2\pi^2} + \sigma E$$
result in

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma} \nabla^2 \mathbf{B} + \frac{2\alpha \mu_5}{\pi \sigma} \nabla \times \mathbf{B}$$  \hspace{1cm} (8)$$

For the field $\mathbf{B}_\pm \sim (\hat{x} \pm \hat{y}) e^{i(kz - \omega t)}$, modes with $0 < k < 2k_*$ are unstable

$$B_k(t) = B_k(0)e^{tk(2k_* - k)/\sigma}, \quad k_* = \frac{\alpha \mu_5}{\pi}$$  \hspace{1cm} (9)$$

The maximally unstable mode occurs for $k = k_*$. 
Using the increase of neutron density due to the electron capture

\[ \Delta n_n \sim 0.1 \text{ fm}^{-3}, \quad (10) \]

Ohnishi and Yamamoto estimated the generated chiral chemical potential

\[ \mu_5 \approx 200 \text{ MeV} \quad (11) \]

that may give enormous magnetic field \( B \sim 10^{18} \text{ G} \). Still electrons have nonzero mass that may hinder the chiral charge generation. Since the electron mass \( m_e = 0.51 \text{ MeV} \) is much less than the electron chemical potential \( \mu_e \approx 100 \text{ MeV} \), it was argued in arXiv:1402.4760 that the electron mass effects can be neglected.
Role of mass


$$\Gamma_m \approx \frac{\alpha^2 m_e^2}{3\pi \mu_e} \approx 1.4 \times 10^{-8} \text{ MeV} \quad (12)$$

Evolution of chiral charge density is governed by

$$\frac{\partial n_5}{\partial t} = n_e \Gamma_w - n_5 \Gamma_m \quad (13)$$

During core collapse the electron fraction changes $\delta Y_e \approx 0.4$ in the free fall time $t_{ff} = 0.1 \text{ s}$ giving the chirality production rate per electron

$$\Gamma_w = \frac{\dot{Y}_e}{Y_e} \sim 1 \text{ s}^{-1} \sim 6.6 \times 10^{-22} \text{ MeV} \quad (14)$$
For the steady state solution, the chiral charge density

\[ n_5 = n_e \left( \frac{\Gamma_w}{\Gamma_m} \right) \sim 10^{-14} n_e \]  

(15)

and the chemical potential

\[ \mu_5 = \frac{\pi^2 n_5}{\mu_e^2} \sim 10^{-14} \mu_e \]  

(16)

are very small.
Nonzero mass strongly (!) hinders the generation of chiral asymmetry.
Jets of active galactic nuclei

Figure: M87 jet, $\gamma \approx 6$, radio lobes stretch up to 80 kiloparsecs
Central engine

Figure: Central engine is supermassive black hole $M = 6.5 \times 10^9 \, M_\odot$ with $R_s = 120 \, \text{AU}$

Magnetic field $B \sim 10^4 \, \text{G}$, time scale $t_0 = R_s/c \sim 10^5 \, \text{s}$ is very large (macroscopic) in view of $R_s \sim 10^{13} \, \text{m}$
For most optimistic $E \sim B$, the chiral charge density due to the chiral anomaly equals naively

$$n_5^{\text{naive}} \simeq \frac{e^2}{2\pi^2} E \cdot B t_0 \sim 10^4 \text{ MeV}^3 \quad (17)$$

The inclusion of the chirality flip changes the situation dramatically with $\Gamma_m = \alpha^2 m_e^2/(3\pi T)$ and $T = 1 \text{ MeV}$

$$n_5 = \frac{e^2}{2\pi^2 \Gamma_m} E \cdot B \sim 10^{-17} \text{ MeV}^3, \quad \mu_5 \sim 10^{-17} \text{ MeV} \quad (18)$$

leading to negligible chiral chemical potential.
Magnetars

Figure: Artist’s conception of a magnetar

$B \sim 10^{11} - 10^{13} \, \text{G} \, (\text{radio pulsars})$, $B \sim 10^{14} - 10^{15} \, \text{G} \, (\text{magnetars})$. About 30 magnetars are known in the Milky Way and are observed as soft gamma-repeaters or anomalous X-ray pulsars. Magnetar SGR 1935+2154 has been associated with fast radio burst
Fast radio bursts

Figure: Artist’s conception of FRB 181112 reaching the Earth

A fast radio burst is a transient radio pulse in the millisecond range with typical frequency 1.4 GHz and releasing on average as much energy as the Sun in 3 days.
Figure: Magnetosphere of a compact star
General properties

- Vacuum model with charges on the compact star’s surface and vacuum outside

- **Corotating** plasma model with the Goldreich–Julian charge density $\rho = \text{div} E \approx -2 \Omega \cdot B$

- Consistency with the Faraday’s law implies the necessity of transient gap regions with $E \cdot B \neq 0$
Chirality production is possible in the gap region, where $E \cdot B \neq 0$.

$B \sim 1/r^3 \rightarrow$ only the polar cap region is of interest for chirality production.

Gap height $h = 3.6 \text{ m}$, voltage drop across the gap $10^{12} \text{ V}$ that gives electric field $eE_{||} = 2.1 \times 10^{-7} \ m_e^2$. 
For magnetar with $B = 10^{15}$ G and plasma temperature $T = 1$ MeV, we find that the steady state solution to

$$\frac{\partial n_5}{\partial t} = \frac{e^2}{2\pi^2} E \cdot B - \Gamma_m n_5$$

leads to a sizeable chiral charge density and chiral chemical potential

$$n_5 = \frac{e^2 EB}{2\pi^2 \Gamma_m} \approx (0.1 \text{ MeV})^3,$$

$$\mu_5 \approx \frac{3n_5}{T^2} \approx 3.5 \times 10^{-3} \text{ MeV}$$

giving

$$k_* = \alpha \mu_5 / \pi = 8 \text{ eV}$$
Dynamics in gap region

- Still electric field $E_{\parallel} = 2.1 \times 10^{-7} \, m_{e}^{2}/e$ is much smaller than the Schwinger electric field $E_{c} = m_{e}^{2}/e$

- Gap is an intermittent phenomenon [D.B. Melrose and R. Yuen, Pulsar electrodynamics, J. Plasma Phys. 82, 635820202 (2016)]

- As $E_{\parallel}$ grows in the charge starvation region, it could lead to avalanches induced by a photon flux

- Gap region opening and closing is a dynamical process → particle-in-cell simulations are necessary

- Our proposition is to include the evolution equation for the chiral charge density and the CME current in these simulations
What should be clarified?

- **Chirality** and electron-positron pair production induced by energetic photons
- The rate of chirality flip $\Gamma_m$ in a superstrong ($|eB| \gg m_e^2$) magnetic field
- Joint evolution of chiral imbalance and magnetic fields
- Inverse magnetic cascade and its observational consequences for electromagnetic emission (relevance for fast radio bursts?)
Inverse magnetic cascade

Figure: Transfer of helicity from shorter to longer modes (red to blue)

Conclusions

- Chirality generation is possible in polar caps of magnetars due to the chiral anomaly.
- Spinodal instability due to the CME leads to strong helical electromagnetic field modes.
- Observational features could be polarized electromagnetic radiation (possibly relevant for fast radio bursts).
Thank you for attention!