Two-flavor color superconductivity in magnetic field*

Lang Yu

Advisor: Igor Shovkovy

Department of Physics, Arizona State University, Tempe, AZ 85287

**Introduction**

- **BCS Mechanism:**

  - Electrons in metals below the critical temperature
  - Cold and dense quark matter

  **BCS mechanism**

  - Superconducting metals
  - Color superconductor
Question:

Does “color superconductivity” exist in nature?
Neutron Stars

- Mass: ~ 1.2 - 2.0 $M_\odot$
- Radius: ~ 10 km
- Central density: ~ $1 \text{ fm}^{-3}$
- Interior temperature:
  - ~ below 10 MeV
    (a few seconds old)
  - ~ 10 keV
    (a few years old)
  - ~ 1 keV
    (a million years old)
  - decrease by neutrino emission
Neutron Stars

- Magnetic fields:
  - $\sim 10^{12} \text{ G (surface of neutron stars)}$
  - $\sim 10^{14}-10^{15} \text{ G, even } 10^{16} \text{ G (surface of magnetars)}$
  - $\sim 10^{18} \text{ G (interiors of magnetars)}$

Color superconductivity

Astrophysical observables
color superconductivity

Quark Cooper pairing:

\[ \langle \psi_i^a \psi_j^b \rangle = P_{ij}^{ab} \Delta \]

- color indices: \( a, b = r, g, b \)
- flavor indices: \( i, j = u, d, s \)
- Dirac indices: \( \alpha, \beta \)
- gap parameter: \( \Delta \)

Two-flavor color superconducting (2SC) phase:

\[ \langle \psi_i^a C \gamma^5 \psi_j^b \rangle \propto \Delta_{2SC} \epsilon^{ab3} \epsilon_{ij3} \gamma^5 \]

Unbroken \( \tilde{U}(1)_{em} \rightarrow \) unbroken rotated photon

\[ \tilde{A}_\mu = \cos \theta A_\mu - \sin \theta A^8_\mu \]

\[ \cos \theta = \frac{g}{\sqrt{g^2 + e^2/3}} \]
The rotated photon $\hat{A}_\mu$ is massless and experiences no Meissner effect.

- The color superconductors can be penetrated by “rotated” magnetic fields.
- Some of Cooper pairs are formed by quarks with opposite rotated charges.

The diquark pairing dynamics is affected by the presence of magnetic fields.
Magnetic 2SC and CFL phases

Recent work on Magnetic 2SC and CFL phases:


The gap parameters are modified by the external magnetic fields.

All studies are performed in the framework of Nambu-Jona-Lasinio (NJL) model with a local four-fermion interaction.
Model

Lagrangian density (2SC phase):

\[
\mathcal{L}_{\text{quarks}}^{em} = \bar{\psi} \left( i \gamma^\mu \partial_\mu - m + \hat{\mu} \gamma^0 + \tilde{e} \gamma^\mu \tilde{A}_\mu \tilde{Q} \right) \psi
\]

Quark spinors:

\[
\psi = \begin{pmatrix}
\psi_{ur} \\
\psi_{ug} \\
\psi_{ub} \\
\psi_{dr} \\
\psi_{dg} \\
\psi_{db}
\end{pmatrix}
\]

Chemical potential matrix:

\[
\mu_{ij,ab} = [\mu \delta_{ij} - \mu_e (Q_f)_{ij}] \delta_{ab} + \frac{2}{\sqrt{3}} \mu_8 \delta_{ij} (T_8)_{ab}
\]

\(\mu_e\) and \(\mu_8\) are introduced because of \(\beta\)-equilibrium and neutrality of quark matter.
Rotated electric charges:

\[ \tilde{Q} = Q_f \otimes I_c - I_f \otimes \left( \frac{T_8}{\sqrt{3}} \right)_c \]

\[
\begin{array}{ccccccc}
    u_r & u_g & u_b & d_r & d_g & d_b \\
    +\frac{1}{2} & +\frac{1}{2} & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\
\end{array}
\]

\[ \tilde{Q} \] charges of different quarks in units of \( \tilde{e} = e \cos \theta \)

Full inverse quark propagators in Nambu-Gorkov basis for \( \tilde{Q} = \pm \frac{1}{2} \):

\[
S_{(\tilde{Q})}^{-1} = \begin{pmatrix}
G_{(\tilde{Q}),0}^{+} & \Delta_{(\tilde{Q})}^{-} \\
\Delta_{(\tilde{Q})}^{+} & G_{(\tilde{Q}),0}^{-}^{-1}
\end{pmatrix}
\]
Schwinger-Dyson equation:

\[
\begin{pmatrix}
\gamma^\mu \\
\end{pmatrix}^{-1}
\] = \begin{pmatrix}
\gamma^\mu \\
\end{pmatrix}^{-1} + \begin{pmatrix}
\gamma^\mu \\
\end{pmatrix}^T
\]

The graphical representation of the Schwinger–Dyson equation in the improved rainbow approximation.

The gap equation in the coordinate space:

\[
\left[ S_{(Q)}^X \right]_{21}^{-1} (u, u') = i g^2 \gamma^\mu (-T^A)^T \left[ S_{(Q)}^X \right]_{21} (u, u') \gamma^\nu T^B D_{\mu\nu}^{AB} (u, u')
\]

The gap equation in terms of Landau levels:

\[
\Delta_m \mathcal{P}_- + \Delta_{m+1} \mathcal{P}_+ = -i \frac{2g^2}{3} \sum_{n=0}^{\infty} \int \frac{d\omega'}{(2\pi)^2} \int \frac{d^2 q_\perp}{(2\pi)^2} \gamma^\mu \Delta_n \left[ \mathcal{L}_{n,m}^{(0)} \frac{\mathcal{E}_n}{\mathcal{C}_n} \mathcal{P}_- + \mathcal{L}_{n-1,m}^{(0)} \frac{\mathcal{E}_n}{\mathcal{C}_n} \mathcal{P}_+ \right] \\
\times \gamma^\nu D_{\mu\nu} (\omega - \omega', k^3 - k'^3; q_\perp),
\]
Gap equation

- **hard-dense-loop approximation:**

\[ \Pi = \begin{array}{c} \text{Hard momentum} \\ \mu \sim K \\ \text{Soft momentum} \\ Q \sim g \mu \end{array} \]

The leading contribution of the gluon self-energy, \( \Pi_{\mu\nu} = (D^{-1})_{\mu\nu} - (D^{-1}_F)_{\mu\nu} \), is proportional to \( g^2 \mu^2 \) in hard-dense-loop approximation.

- **Gluon propagator in Coulomb gauge:**

\[
D_{\mu\nu}(Q) = -\frac{Q^2}{q^2} \delta_{\mu0} \delta_{\nu0} - \frac{P^T_{\mu\nu}}{Q^2 - F} \quad \frac{P^T_{\mu\nu}}{Q^2 - G}
\]

\[
P^T_{00} = P^T_{0i} = 0, \quad P^T_{ij} = \delta_{ij} - \hat{q}_i \hat{q}_j.
\]

In the most important regime, \( q^0 \ll |\vec{q}| \ll m_D \),

\[
F \simeq m_D^2, \quad G \simeq \pi \frac{m_D^2 q^0}{|\vec{q}|}, \quad \text{where} \quad m_D^2 = \left(\frac{g\mu}{\pi}\right)^2
\]

Long-range interaction for the gluons are cut off by Debye screening and Landau damping.
In the weak field limit, the difference between the neighboring levels is vanishingly small.

Gap equation

Quark propagator in the weak magnetic field limit:

\[ \bar{S}_{(+{\frac{1}{2}})}^{X}(i\omega_{E}, k^{3}, k_{\perp}) = i\gamma^{5} \Delta \left[ K^{(0)} + K^{(1)} + K^{(2)} \right] \]

\[ K^{(0)} \sim \text{independent of } \tilde{B} \]

\[ K^{(1)} \propto \tilde{e}\tilde{Q}\tilde{B} \]

\[ K^{(2)} \propto (\tilde{e}\tilde{Q}\tilde{B})^{2} \]
Gap equation in the weak magnetic field limit:

$$\Delta(\omega) = T^{(0)}(\omega) + T^{(1)}(\omega) + T^{(2)}(\omega)$$

$$T^{(i)}(\omega) = -\frac{2g^2}{3} \int \frac{d\omega'}{2\pi} \int \frac{d^3k'}{(2\pi)^3} \Delta(\omega') \gamma^\mu K^{(i)}(\omega', k') \gamma'' D_{\mu\nu}(\omega - \omega', k - k')$$

The 0th order contribution:

$$\Delta^{(0)} \simeq \Lambda \exp\left(-\frac{3\pi^2}{\sqrt{2g}} + 1\right)$$

where

$$\Lambda = \frac{4(2\mu)^3}{(\pi m_D^2)}$$
The 2nd order correction:

\[ \Delta^{(B)} \simeq \Lambda \exp \left( -\frac{3\pi^2}{\sqrt{2}g_{\text{eff}}} + 1 \right) \simeq \Delta^{(0)} e^{\beta_{B_k}} \]

where

\[ \beta_{B_k} = \frac{81\pi^3 (\tilde{e}\tilde{B})^2}{4\sqrt{2}g^3 \bar{\mu}^4} \sin^2 \theta_{B_k} \]

\[ g^2 \rightarrow g_{\text{eff}}^2 = g^2 \left( 1 + \frac{27\pi (\tilde{e}\tilde{B})^2}{2g^2 \bar{\mu}^4} \sin^2 \theta_{B_k} \right) \]
The validity of the weak field approximation requires: \[ |\tilde{e}\tilde{B}|^2 \lesssim g^2 \bar{\mu}^4 \]

The magnetic field correction to the gap:

- The correction appears to be very small,
  \[ \beta_{Bk}^{(\text{max})} \approx 1.3 \times 10^{-2} \left( \frac{400 \text{ MeV}}{\bar{\mu}} \right)^4 \left( \frac{\tilde{B}}{10^{18} \text{ G}} \right)^2 \]
  - The correction is proportional to \( \sin^2 \theta_{Bk} \).
  \[ \beta_{Bk} = \frac{81\pi^3 (\tilde{e}\tilde{B})^2}{4\sqrt{2}g^3 \bar{\mu}^4} \sin^2 \theta_{Bk} \]
Here we use Lowest Landau level (LLL) approximation and the same gluon propagator used in weak magnetic fields.

**Gap equation in the strong magnetic field limit:**

\[
\Delta^{(B)}(\omega_E) \approx \frac{g^2}{72\pi^2} \int_{-\infty}^{+\infty} d\omega'_E \frac{\Delta^{(B)}(\omega'_E)}{\sqrt{(\omega'_E)^2 + (\Delta^{(B)})^2}} \ln \frac{\Lambda_B}{|\omega'_E - \omega_E|}
\]

Here we use Lowest Landau level (LLL) approximation and the same gluon propagator used in weak magnetic fields.

**The Gap estimation:**

\[
\Delta^{(B)} = \frac{4\pi |\tilde{e}\tilde{B}|^{3/2}}{g^2 \bar{\mu}^2} \exp\left(-\frac{3\pi^2}{g} + 1\right)
\]

This result shows that the strong magnetic field strengthens the diquark pair formation (no directional dependence).
Cold and dense quark matter is expected to be a color superconductor, and may exist in the interiors of neutron stars.

We derive a general gap equation for an arbitrary magnetic field in the 2SC phase in terms of Landau levels.

In weak magnetic field limit, the gap function shows a directional dependence in the momentum space and the magnetic field strengthens the gap parameter slightly.

In strong magnetic field limit, the magnetic fields enhance quark-quark Cooper pairing (no directional dependence).
Summary & outlook

The numerical computations of the gap equation can be performed for an arbitrary magnetic field.

The effects of different quark masses and chemical potentials can be investigated.

Extend our understanding of dense quark matter by clarifying possible directional dependences of the gap function and the evolution of such a dependence between the two limiting cases.

Study the consequences of magnetic 2SC phase on the observable properties of neutron stars.
Thank you