Neutrino cooling and spin-down of rapidly rotating neutron stars

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My collaborators on this & related work

Work in Progress:
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(PRD 78, 123007 (2008))
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Outline

- Spin-down of neutron stars: Mechanisms
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- $r$-modes: Gravitational wave emission
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• **Spin-down of neutron stars**: Mechanisms

• \( r \)-**modes**: Gravitational wave emission

• **Viscosity**: \( r \)-mode damping
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• **Spin-down of neutron stars**: Mechanisms
  
  • $r$-modes: Gravitational wave emission
  
  • **Viscosity**: $r$-mode damping
  
  • **Neutrino Cooling**: Effects of the $r$-mode perturbation
Braking radiation

Rotating dipole model

- **spin-down**
  \[
  \dot{E} = -\frac{B^2 \sin^2 \alpha \Omega^4 R^6}{6c^3} = I\Omega \dot{\Omega}
  \]

- **Braking index**
  \[
  \dot{\Omega} = K\Omega^n; \quad n = \frac{\Omega \dot{\Omega}}{\Omega^2} = 3
  \]

- **Magnetic field**
  \[
  \Omega = \frac{2\pi}{P}; \quad B = \left(\frac{3Ic^3 P\dot{P}}{2\pi^2 R^6}\right)^{1/2}
  \]

- **Characteristic age**
  \[
  \tau \approx \frac{-\Omega}{(n-1)\Omega} = \frac{P}{2P}
  \]

Radio emission is a tiny fraction of total radiation.
Rotation rates

Supernova simulations predict rapid rotation rates $P \sim 0.2\text{ms}$

Observed rotation rates are smaller ($P \sim 0.5\text{s}$)

Estimated surface field:

$$B_0 \approx 3.2 \times 10^{19} \sqrt{P \dot{P}} \text{ G}$$

Characteristic age:

$$t_c = \frac{P}{2 \dot{P}}$$

"braking index" -1

"typical pulsar" $P=0.5\text{s}$ $B=10^{12} \text{ G}$
Gravitational wave emission

\[ \frac{dJ}{dt} = -I \Omega F_{\text{mag}} + \dot{J}_a(t) - 2K \alpha^2 I \Omega F_g \]

\[ F_g = \frac{1}{47} M_{1.4} R_{10}^4 P_{-3}^{-6} \, \text{s}^{-1} \]


- Gravitationally driven spin-down can be important during the early cooling stages of an isolated neutron star or “spun-up” millisecond pulsars
Stellar Oscillations

Oscillation modes are classified by nature of restoring force

\( r \)-modes: Coriolis force \( (\vec{\Omega} \times \vec{v}) \) term in rotating stars

\( p \)-modes: Pressure fluctuations, convective instability

\( p \)-modes used in helioseismography; verified solar model.
Fluid perturbation equations

(perturbed) Variables:

Energy density, Pressure \( \delta \rho, \delta P \)

Velocity \( \delta \vec{v} \)

Grav. Potential \( \delta \Phi \)
Fluid perturbation equations

(perturbed) Variables:

- Energy density, Pressure: $\delta \rho, \delta P$
- Velocity: $\delta \vec{v}$
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obey equations:

- Perturbed Euler equation: linearized
- Poisson equation:
  $$\delta (\nabla^2 \Phi + 4\pi G \rho) = 0$$
- Continuity equation:
  $$\delta (\partial_\mu j^\mu) = 0$$
Fluid perturbation equations

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\]

- Poisson equation

- Continuity equation

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\delta ( \partial_\mu j^\mu ) = 0
\]

Specify barotropic Equation of State \(( P = P(\rho) )\)
Visualizing $r$-modes

- The angular dependence of the flow (latitude dependence) is given by magnetic-type vector spherical harmonics:

$$\vec{Y}_l^B = [l(l + 1)]^{-1/2} r \nabla \times (r \nabla Y_{ll})$$

- Flow of fluid element in $r$-mode conserves vorticity

$$\frac{d}{dt} \left( \hat{e}_r (\nabla \times \delta \vec{v}) + 2 \hat{e}_r \vec{\Omega} \right) = 0$$

$r$-mode frequency

$(r$-mode freq./rotation freq.) vs. rotation freq. (Kepler units)

Leading order in $\Omega$

$n = \text{polytropic index (neutron star) of mass } 1.4M_{\text{sun}}, R=12.5\text{km}$

Quark matter parameters ($B^{1/4}=165\text{ MeV, } m_s=150\text{ MeV}$)
(mass $1.2M_{\text{sun}}, R=9.7\text{km}$)
$r$-mode instability

Inertial observer measures effective frequency

$$\omega_r^{(in)} = \omega_r^{\text{rot}} + m\Omega = \frac{(m - 1)(m + 2)}{(m + 1)}\Omega$$

For $m \geq 2$, a prograde mode in the inertial frame appears retrograde in the rotating frame
$r$-mode timescales

The timescale associated to growth or dissipation ($\tau$) is given by

$$\frac{1}{\tau_i} = -\frac{1}{E} \left( \frac{dE}{dt} \right)_i ; \quad i = GW, \zeta, \eta$$
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$r$-mode timescales

$\tau_{\zeta,\eta} \gg \tau_{GW}$: $r$-modes will be effective in spinning down the star; spins slowly

$\tau_{\zeta,\eta} \ll \tau_{GW}$: $r$-modes are quickly damped; star can spin rapidly!
Critical rotation frequency

At the critical frequency $\Omega_c$, fraction of energy dissipated/unit time exactly cancels against $r$-mode growth by gravitational wave emission.

$$\frac{1}{\tau(\Omega_c)} = \left[ \frac{1}{\tau_\zeta} + \frac{1}{\tau_\eta} + \frac{1}{\tau_{GW}} \right] (\Omega_c) = 0$$
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Signal duration

- Determine the set of rotation frequencies where gravitational waves are undamped

- Determine how much time the Star spends in this region

Graduate student - Stou Sandalski
Evolution equations

\[ t_{\text{dur}} = \int_{\Omega_i}^{\Omega_f} \frac{d\Omega}{\dot{\Omega}} \]
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\[ \dot{\Omega} = -\frac{2\Omega}{\tau_V} \frac{\alpha^2 Q}{1 + \alpha^2 Q} \]

\[ \dot{\alpha} = \frac{-\alpha}{\tau_{\text{GR}}} - \frac{\alpha}{\tau_V} \frac{1 - \alpha^2 Q}{1 + \alpha^2 Q} \]

\( \alpha \) is \( r \)-mode amplitude, \( Q = \frac{3\bar{J}}{2\bar{I}} \)
Temperature Profile

\[ C_v \frac{dT}{dt} = -\epsilon_\nu + \epsilon_h \]

- \( C_v \) is the specific heat
- \( \epsilon_\nu \) is the neutrino emissivity and
- \( \epsilon_h \) is the chemical heating rate
Revisiting assumptions

So far, studies have used equilibrium emissivities

We will explore effects of non-equilibrium emissivities

Effect of \( r \)-mode

(deviation from equilibrium)

- \[
\frac{\delta \bar{n}_B}{n_B} = R^2 \Omega^2 \frac{d\rho}{dP} \left[ \delta U_0 + \delta \Phi_0 \right]
\]
- \[
\delta U_0 = \alpha \left( \frac{r}{R} \right)^{m+1} P_{m+1}^m (\cos \theta) e^{im\phi}
\]
- \[
\delta \Phi_0 = \alpha \delta \phi_0 (R) P_{m+1}^m (\cos \theta) e^{im\phi}
\]

Re-equilibration:

\[
\delta \mu_i = A_i \delta X_s + B_i \delta X_e + C_i \left( \frac{\delta \bar{n}_B}{n_B} \right) ; \quad X_s = \frac{n_s}{n_B}, \quad X_e = \frac{n_e}{n_B}
\]
Application: \((u, d, s)\) quark matter

\[
\frac{\partial e^{(i)}_{\text{tot}}}{\partial \delta \mu_i} = 3(\Gamma_f^{(i)} - \Gamma_b^{(i)}) = 3\Gamma_{\text{net}}^{(i)}; \quad i = 1, 2
\]

\[
\Gamma_{\text{net}}^{(1)} = \Gamma_{d \rightarrow u + e^- + \nu_e} - \Gamma_{u + e^- \rightarrow d + \nu_e} = \lambda_1 \delta \mu_1
\]

\[
\Gamma_{\text{net}}^{(2)} = \Gamma_{s \rightarrow u + e^- + \nu_e} - \Gamma_{u + e^- \rightarrow s + \nu_e} = \lambda_2 \delta \mu_2;
\]

\[
\Gamma_{\text{net}}^{(3)} = \Gamma_{u + d \rightarrow s + u} - \Gamma_{s + u \rightarrow u + d} = \lambda_3 (\delta \mu_1 - \delta \mu_2) = \lambda_3 \delta \mu_3
\]

- Same \(\lambda_i\)s are used to compute the bulk viscosity
- \(\lambda_{1,2}\) are Urca processes that become as important as \(\lambda_3\) for \(0.1 < T < 1\) MeV and \(\omega_r \leq 1\) kHz

(Shovkovy et al., PRD 75, 125004 (2007))
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- We expect faster cooling; Evaluate effects on $t_{\text{dur}}$ (shorter, stronger?)